AE525 Research Project
Aerodynamic Heating with Turbulent Flows

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Abstract

This paper discusses approximate methods which have been used to calculate turbulent heating rates for a flat-plate case. Simple approximate methods are needed for a practical engineering analysis because of their short computing-time. Although solutions by computational fluid dynamics (CFD) would give the most precise prediction, CFD is still costly in terms of its long computing-time. The paper especially deals with turbulent flows. The matter of turbulent flows is critical for aerodynamic heating since the heating rate in turbulent flows are much higher than that in laminar flows. As shall be seen, the methods for turbulent heating are highly empirical owing to the lack of knowledge of the turbulent mechanism. However, it will be shown that those methods has been successfully used as practical tools. The paper begins with the basic concept of calculation procedure of aerodynamic heating. A presentation of three major engineering methods and discussion follows. The paper concludes with the prospect of approximate methods.

1 Introduction

Aerodynamic heating is the heating of an object in a very high-speed flow due to compression and friction within the boundary layer around the object. This matter is extremely important for high-speed vehicle disign in terms of the thermal protection of a vehicle. In general, the flow around such a vehicle is likely to become turbulent due to the high Reynolds number, which strongly affects the thermal environment of the vehicle. As can be seen in figure 1, for a given M_e and Re, the turbulent values of the Stanton Number, C_H, are considerably larger than the laminar counterparts, which demonstrates the importance of predicting turbulent flows.

Figure 1. Stanton number as a function of Reynolds number and Much number for an insulated plate (using Van Driest II for turbulent cases, [1])

The approach to determine the aerodynamic heating environment may range from simple approximate analyses to detailed CFD solutions and actual flight tests. As the design process progresses, increasing levels of detail are required. For a preliminary analysis, computing-time economy is very important. Short computing-time permits the method to be applied to such tasks as parametric studies, conceptual vehicle design, and trajectory optimization where many passes through the program may be required. Therefore, CFD solutions, although feasible , are not yet practical for engineering analyses and design in terms of the long computing-time. On the other hand, approximate methods, although not as accurate as CFD, have this advantage and have been used as useful tools.

For slender sharp-nosed bodies, the heat transfer coefficients are frequently approximated by using the comparable flat-plate values, while for blunt bodies, stagnation-point values are used. For turbulent flows, flat-plate cases are more significant rather than stagnation-point cases since flows are unlikely to become turbulent in the latter case due to the low Reynolds number there. Therefore, the paper focuses on a flat-plate case. The basic formulation begins with the definition of the heat transfer coefficient. There are, in fact, two local heat transfer coefficients; the Nusselt Number, N_u, and the Stanton Number, C_H, which are connected via $N_u = C_H Re Pr$. Let us use the Stanton number here. The Stanton Number is defined as

$$C_H = \frac{q_w}{\rho_e u_e (h_{aw} - h_w)} \tag{1}$$

where q_w is the local heat flux (energy per second per unit area) to the surface, ρ_e and u_e are the density and velocity at the outer edge of the boundary layer , respectively, and h_{aw} and h_w are the adiabatic wall enthalpy (recovery enthalpy or equilibrium enthalpy) and the enthalpy at the surface respectively. For a flat-plate with zero attack angle, quantities with subscript "e" can be considered as constant values. The physical meaning of the Stanton number is obvious from equation(1), i.e., the Stanton Number gives the fraction of the energy flux that will go into the aerodynamic surface. From the definition, the heat flux is given by

$$q_w = \rho_e u_e C_H (h_{aw} - h_w) \tag{2}$$

In most engineering calculations, the values of h_{aw} is expressed in terms of the recovery factor, r, defined as

$$h_{aw} = h_e + r\frac{{u_e}^2}{2} \tag{3}$$

where h_e is the enthalpy at outer edge of boundary layer. As can be seen, r denotes the fraction of kinetic energy converted into thermal energy within the boundary layer. In terms of r, q_w is written as

$$q_w = \rho_e u_e C_H (h_e + r\frac{{u_e}^2}{2} - h_w) \tag{4}$$

For a calorically perfect gas, where $h = c_p T$, it can be written as

$$q_w = \rho_e u_e C_H c_p [T_e(1 + r\frac{\gamma - 1}{2}{M_e}^2) - T_w] \tag{5}$$

Therefore, it is necessary to know C_H and r in order to compute q_w regardless of whether a flow is laminar or turbulent.

2 Results and Discussion

For laminar boundary layers, exact solutions for r and C_H could be obtained by solving the equations which model the viscous flow over important regions of a vehicle. In contrast with laminar boundary layers, there is no complete theory for modeling the turbulent boundary layer. The equations for turbulent boundary layers can be formulated only with the aid of experimental data. Therefore, all turbulent theories are semiempirical.

2.1 The recovery facor and the Stanton number for turbulent flows

It is possible to obtain a general expression for the recovery factor from the turbulent boundary layer equations in terms of shear stress which must be modeled [1]. However, the approximation in the form

$$r = \sqrt[3]{Pr} \tag{6}$$

has been widely used, where Pr represents the laminar Prandtl number ($Pr \approx 0.71$ for air). Figure 2 shows measured values of r at different Mach numbers and Reynolds numbers. It is seen that $\sqrt[3]{Pr} = \sqrt[3]{0.71} = 0.892$ is a good approximation.

Figure 2. Recovery factors in terms of Reynolds number from measurements on cones at freestream Mach numbers $M_\infty = 1.2$ to $M_\infty = 6.0$ [4]

Table 1. The conditions under which the data in figure 2 was obtained [4]

This formula was first introduced as an empirical correlation. However, later, Dorrance [5] derived this approximate formula from compressible turbulent-boundary-layer equations by breaking the boundary layer into two parts; a laminar sublayer (inner layer) and an outer turbulent portion (outer layer). Dorrance first derived the following expression.

$$r = Pr_T + (Pr - Pr_T)(\frac{u_L}{u_e})^2$$

where u_L is the velocity at the interface of the two layers and Pr_T represents the turbulent Prandtl number. Then, it is assumed that Pr_T=1 and $(\frac{u_L}{u_e})^2 \approx$ 0.33 to get

$$r = 1 + 0.33(Pr - 1) \simeq Pr^{1/3}$$

which is, if $Pr = 0.71$, correct with $\pm 10\%$ for $\frac{u_L}{u_e}$ varying from 0.22 to 0.83, i.e., for most values of $\frac{u_L}{u_e}$ of concern to us. Therefore, if $\frac{u_L}{u_e}$ is outside these ranges, exponents other than 1/3 may be chosen.

As for the Stanton number, almost all approximate turbulent heat transfer methods are primarily based on equating the skin friction to the Stanton number through the Reynolds analogy. Therefore, once the local friction coefficient is known, the local Stanton number is directly obtained. Any refinement in this procedure would require further knowledge of the turbulence mechanism. The general form of Reynolds analogy is written as

$$C_H = \frac{c_f}{2s} \tag{7}$$

where c_f is the local skin friction coefficient and s is called the Reynolds analogy factor. In a mannar similar to the recovery factor, it is also possible to obtain a general expression for the Reynolds analogy factor [1] . Nevertheless, in most cases, it is approximated by Colburn's version of Reynolds analogy;

$$C_H = \frac{c_f}{2P_r^{\frac{2}{3}}} \tag{8}$$

, which was originated for flat plate incompressible laminar flows, although some variations exist. At least, one other version is von Karman's version of Reynolds analogy;

$$C_H = \frac{c_f}{2[1 + 5\sqrt{\frac{c_f}{2}}(Pr - 1) + ln[1 + \frac{5}{6}(Pr - 1)]]} \tag{9}$$

where it is assumed that Pr_T=1. Van Driest compared these formulae with the general expression that he derived [1] . The comparison showed that Colburn's estimate was a good approximation for a turbulent boundary layer on a flat plate in air.

In order to compute C_H, it is now necessary to obtain c_f. In general, a method of obtaining c_f corresponds to an approximate method for the turbulent heating rates. Three of the most popular methods are presented in this paper; the Eckert's reference enthalpy method, Van Driest II, and The Spalding and Chi method.

2.2 Eckert's reference enthalpy method

The reference temperature methods was originated by Rubesin and Johnson (1949) for laminar boundary layers and was applied to turbulent boundary layers with modified coefficients by Sommer and Short (1955). The basic assumption is that a temperature within the boundary layer can be calculated , using an expression with empirically determined coefficients, which will yield the correct skin friction in a compressible boundary layer when incompressible relations are used. Eckert [6] extended the method to high-speed flows in equilibrium having real gas effects by using enthalpy in lieu of temperature. Although this method was originated as an empirical correlation, later Dorrance [5] showed that the reference temperature was a direct consequence of the similarity relations for compressible laminar flow. The common expression for the reference value of the enthalpy (the Eckert version) is

$$h^* = 0.22h_{aw} + 0.28h_e + 0.5h_w \tag{10}$$

although a number of variations have been used. For turbulent flows over a flat plate , the local skin friction can be obtained by

- For $Re^* \leq 10^7$, the Blasius incompressible, turbulent skin friction relation:

$$\frac{c_f}{2} = \frac{0.0296}{(Re^*)^{0.2}} \tag{11}$$

- For $Re^* > 10^7$, the Schults-Grunow skin friction relation:

$$\frac{c_f}{2} = \frac{0.185}{(log_{10}Re^*)^{2.584}} \tag{12}$$

where Re is the Reynolds number defined as $Re = \rho_e u_e x/\mu_e$ (x:geometrical coordinate along a plate with origin at leading edge). The asterisk denotes property evaluation at the reference enthalpy given by equation(10). In addition, other fluid properties such as ρ_e and Pr have to be evaluated at the reference enthalpy, i.e., have to be replaced by ${\rho_e}^*$ and Pr^* in equations (5) and (6). In this procedure, an iterative calculation is usually necessary since r and h_{aw} depend on h^*, which itself depends on r and h_{aw} .

2.3 Van Driest II

Van Driest II refers to Van Driest's second method which uses the von Karman mixing length. In addition, the Crocco temperature profile and Pr_T=Pr=1 are assumed through the boundary layer. Van Driest obtained the following formula as a semianalytical solution of the turbulent boundary layer equations in compressible fluids with zero pressure gradient [2] .

$$\frac{0.242}{c_f^{1/2}(\frac{\gamma-1}{2}{M_e}^2)^{1/2}}(\arcsin\alpha + \arcsin\beta) = 0.41 + \log(Re \cdot c_f \cdot \frac{\mu_e}{\mu_w}) \tag{13}$$

where

$$\begin{aligned}
\alpha &= (2A^2 - B)/(B^2 + 4A^2)^{1/2} \\
\beta &= B/(B^2 + 4A^2)^{1/2} \\
A^2 &= (\frac{\gamma-1}{2}{M_e}^2)/(\frac{T_w}{T_e}) \\
B &= (1 + \frac{\gamma-1}{2}{M_e}^2)/(\frac{T_w}{T_e})
\end{aligned}$$

and $\mu \propto T^\omega$ can be assumed ($\omega \approx 0.67$ for air). It should be kept in mind that a proper value of Pr should be used in equation(6) although $Pr = 1$ has been assumed.

2.4 The Spalding and Chi method

This method is based on Van Driest II. Spalding and Chi considered equation(13) as an incompressible formula which has

been stretched by compressibility and viscosity effects. Thus equation(13) can be written as

$$c_f = \frac{1}{F_C} c_{f\,inc}(ReF_{Rx}) = \frac{1}{F_C} c_{f\,inc}(Re_\theta F_{Rx} F_C) \tag{14}$$

where

$$F_C = \frac{T_{aw}/T_e - 1}{(\arcsin\alpha + \arcsin\beta)} \tag{15}$$

$$F_{Rx} F_C = \frac{\mu_e}{\mu_w} \tag{16}$$

and Re_θ is the Reynolds number based on the momentum thickness. This form may be called a compressible transformation. In fact, equation(13) reduces to an incompressible relation when $M_e \approx 0$ is assumed [13]. In addition, Spalding and Chi suggested a new correlation function in place of equation(16);

$$F_{Rx} F_C = \frac{1}{(T_w/T_e)^{0.702}(T_w/T_{aw})^{0.772}} \tag{17}$$

which was empirically determined by using a large body of data. The procedure is, therefore, as follows. First, one computes F_{Rx} and F_C to get ReF_{Rx} or $Re_\theta F_{Rx} F_C$ and then uses one's favorite incompressible skin friction formula to compute $c_{f\,inc}$, dividing the result by F_C to obtain the desired compressible c_f.

2.5 Discussion

The ability of some methods to predict turbulent heat transfer has been compared by several authors. For instance, Hopkins and Inouye [7] compared the three methods presented here and the method of Coles (1964). They concluded that the Van Driest II give the best overall results. On the other hand, the comparison by Zoby and Grave [9] showed that the best overall agreement was obtained with the Spalding and Chin method and Eckert's reference enthalpy method with the Schultz-Grunow relation, using the Colburn's Reynolds analogy. This conclusion suggests that a certain combination of the formulae give a better prediction. In these comparisons, however, disagreement between a method and data of 15 $\sim$ 20 % is common. In reference[17], Chien measured turbulent skin-friction coefficients and heat transfer distributions on an axisymmetric sharp cone at $M_\infty = 7.90$, and compared the data with several methods. The comparisons by Chien poses one significant problem.

Figure 3. Stanton number distribution: a) $T_w/T_0 = 0.35$; b) $T_w/T_0 = 0.20$; c) $T_w/T_0 = 0.11$ [17]

As can be seen in figure 3, the degree of agreement between the turbulent data and the methods depends strongly on T_w/T_0 (wall-to-stagnation temperature ratio). The conclusion drawn from the figure in [17] is that if von Karman version of Reynolds analogy is utilized, Van Driest II gives reasonable predictions of heat transfer for $T_w/T_0 > 0.2$, whereas only the method of Spalding and Chi yields a value that is within 10 percent of the measurement at $T_w/T_0 = 0.11$, and that the wall-temperature effect on the Stanton number is quite small for $T_w/T_0 > 0.2$. Therefore, it implies that almost all methods cannot be used for low-wall temperatures with a high level of confidence.

In practice, for instance, the real-time heating algorithm [10], installed on the Ames Research Center Dryden Flight Research Facility real-time flight simulator, uses Van Driest II with Colburn's version of Reynolds analogy to compute turbulent heat trsnsfer coefficients, with good accuracy. In addition, in 1993, several engineering methods and CFD predictions, as well as wind-tunnel test results, were compared as a preliminary analysis of the heating environment of the HL-20 by Wurster and Stone [11]. As an engineering method for turbulent heating along the windward centerline and the windward surface of the wings, Eckert's reference enthalpy method coupled with the Schultz-Grunow skin friction relation was used in conjunction with the Mangler transformation. The comparison showed very good agreement and demonstrated the validity of the engineering method.

3 Concluding Remarks

Starting with the basic concept of aerodynamic heating, the paper discussed two major approximate methods. Despite the lack of knowledge of turbulent mechanism, these methods appear to give rather accurate predictions. However, it must be kept in mind that these are approximate methods which inevitably contain a degree of error. It seems that the improvement primarily depends on the understanding of the turbulent properties, i.e., the eddy viscosity and the turbulent Prandtl number, since the recovery factor and the Stanton number are basically derived from the boundary layer equations containing those quantities. Again, it should be noted that as is often the case with turbulent related problems, the approximate methods do not necessarily give accurate predictions for every problem. Therefore, care must be taken when they are applied beyond the range of conditions for which they were derived or proved to be valid. In further research, more systematic investigations especially for low-wall temperatures are necessary, which is undoubtedly the great significance in the optimal design of high-speed vehicles. Until the day comes when CFD can be more practical or the boundary layer can be controlled and held laminar, these approximate methods, however, are still essential tools for predicting turbulent aerodynamic heating rates.

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