

Aero 420 Fall 1998
Problem Set No. 4 Solutions
Average Grade: —

This document is a part of the model solutions to a problem set in an aerodynamics class.

Problem 2 was to criticize the popular theory of flight which goes like this:

An airfoil produces lift because the upper surface is curved so that air flowing over the upper surface speeds up to meet air flowing the lower surface at the trailing edge. The faster air speed causes a lower pressure on the upper surface due to the Bernoulli principle, thus producing a lift force.

The model solution was written by Hiroaki Nishikawa who was then the teaching assistant to the class. The formula for the lift, Equation 9, was derived from the popular theory of flight by Hiroaki Nishikawa.

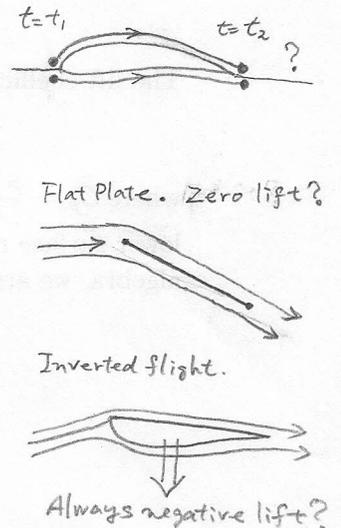
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2 Problem 2

Consider two imaginary particles (massless and volumeless) which have been convected together from the upstream and just arrived at the front stagnation point on an airfoil. Suppose that one travels on the upper surface and the other on the lower surface towards the trailing edge. Now the theory tacitly assumes that they arrive at the trailing edge at the same time. Do they really meet again at the trailing edge? In fact, there is no reason why they should. This is one of the most fallacious assumptions of the theory. You might want to check it by using the applet.

Second, the theory assumes that stagnation points are always located at the leading and trailing edges. But then it leads us to the conclusion that a flat plate cannot generate a lift at any angle of attack because the upper path and the lower path are always the same. Besides, an inverted level flight is impossible because it always generates negative lift. However we all know that these are all possible, i.e.



a flat plate generates lift and an aircraft can fly with an inverted position. These are the examples of observable effects that the theory does not explain.

Now let us consider the time for a particle to go from the leading edge to the trailing edge. According to the theory, the leading edge is a stagnation point. In the neighborhood of a stagnation point, we know that the speed is proportional to the distance(cf. Problem set No.1).

$$q = ks \quad (3)$$

where q is a flow speed, k is some proportionality constant, and s is the normalized distance($s = 0$ at LE, $s = 1$ at from the stagnation point along the surface(either upper or lower)). The time t for the particle to reach the trailing edge can be computed by integration.

$$t = \int_{LE}^{TE} \frac{ds}{q} = \frac{1}{k} \int_0^1 \frac{ds}{s} = \infty. \quad (4)$$

Therefore the particle can never reach the TE. This is in fact a valid conclusion, but remember that almost no particles do pass through the stagnation point. What we see is that because it would take infinite time to go from one stagnation point to the other, there is nothing useful obtained by considering the question.

The prospect of obtaining numerical information is not zero. We can obtain an estimate of the lift force of an airfoil as follows. Suppose that the angle of attack is zero so that the stagnation point is located at the leading edge. Since the theory does not tell us how air speed changes along the surfaces, we may use average speeds of air on the upper and lower surfaces, V_u and V_l , defined by

$$V_u = \frac{L_u}{t}, \quad V_l = \frac{L_l}{t} \quad (5)$$

where L_u and L_l are the length of the upper and the lower surfaces respectively, and t is the time for air to pass along the airfoil. According to the theory, the time t must be the same, and we have

$$\frac{L_u}{V_u} = \frac{L_l}{V_l}. \quad (6)$$

In addition, the theory claims that the time t is independent of the path. We can then assume that $t = \frac{c}{V_\infty}$ where c is the chord of the airfoil(in the case of a flat lower surface). Therefore we have

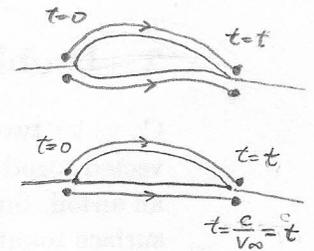
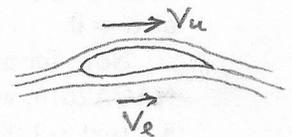
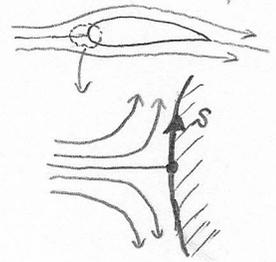
$$\frac{L_u}{V_u} = \frac{L_l}{V_l} = \frac{c}{V_\infty}. \quad (7)$$

The lift coefficient C_l is given by

$$C_l = C_{pl} - C_{pu} \quad (8)$$

where $C_p = \frac{P - P_\infty}{\frac{1}{2}\rho V_\infty^2}$ by Bernoulli's equation, and subscripts u and l denote upper and lower surface respectively. Using the results from the theory, i.e. Eq(7), after some algebra, we arrive at the following estimate of the lift coefficient

$$C_l = \left(\frac{L_u}{c}\right)^2 - \left(\frac{L_l}{c}\right)^2. \quad (9)$$

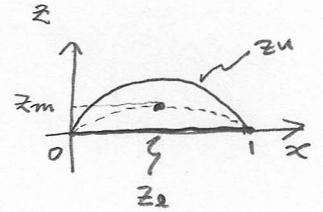


The formula actually applies to all angles of attack because the theory assumes that the front stagnation point is always at the leading edge. It then implies that the lift is independent of the angle of attack, which is wrong.

This formula gives the estimate for C_l at zero angle of attack for a given airfoil with its length L_u and L_l . But now the question “How accurate is this?” should be raised.

Consider a simple case that we are familiar with. Suppose that the lower surface is just a horizontal line ($z_l = 0$) and the upper surface is a parabola defined by

$$z_u(x) = 8z_m x(1-x) \quad (10)$$



where z_m is the maximum ordinate of the camber line, which is assumed to be very small, and z and x have been nondimensionalized by the chord length ($c = 1$, $x = 0$ at LE and $x = 1$ at TE). The mean camber line $z(x)$ is then given by

$$z(x) = \frac{1}{2}(z_u(x) + z_l(x)) = 4z_m x(1-x). \quad (11)$$

According to thin-airfoil theory (Example(5.1) in the text(p.148)), C_l at zero angle of attack for the parabolic thin-airfoil is given by

$$C_l = 4\pi z_m, \quad (12)$$

i.e. C_l is proportional to z_m .

Now we derive a formula for C_l from the “theory of flight”. For the airfoil under consideration, we have $L_l = 1$ and thus need to compute L_u only. But L_u is simply an arc-length of $z_u(x)$, and therefore given by

$$L_u = \int_0^1 \sqrt{1 + \left(\frac{dz}{dx}\right)^2} dx. \quad (13)$$

Since $\frac{dz}{dx}$ is assumed to be very small, we may expand the integrand to get

$$L_u = \int_0^1 \left(1 + \frac{1}{2} \left(\frac{dz}{dx}\right)^2 + O\left(\left(\frac{dz}{dx}\right)^4\right) \right) dx. \quad (14)$$

We neglect the higher order terms and carry out the integral.

$$L_u = \int_0^1 \left(1 + \frac{1}{2} \left(\frac{dz}{dx}\right)^2 \right) dx = \int_0^1 \left(1 + \frac{1}{2} (4z_m)^2 (-2x+1)^2 \right) dx = 1 + \frac{1}{6} z_m^2. \quad (15)$$

Substituting this into Eq(9), we finally obtain C_l by the “theory of flight” as

$$C_l = \frac{1}{3} z_m^2 \quad (16)$$

where a higher order term (z_m^4) has been neglected again. This formula says that C_l is proportional to z_m^2 . But this is wrong as it should be proportional to z_m . Therefore there is something wrong in the “theory of flight” as discussed earlier, and although we did obtain an estimate of lift it was very poor and greatly underestimates the lift because z_m is usually very small. Actually, before the beginning of this century, arguments like this were used to prove the impossibility of heavier-than-air flight.