

## Osher's Approximate Riemann Solver (1D Euler) (Nishikawa, Dec. 1998)

$$F_{j+1/2} = f(u_j) + \int_j^{j+1} A(u) du = f(u_{j+1}) - \int_j^{j+1} A^+(u) du$$

$$\text{or } F_{j+1/2} = \frac{1}{2} [f(u_j) + f(u_{j+1})] - \frac{1}{2} \left[ \int_j^{j+1} A^+(u) du - \int_j^{j+1} A^-(u) du \right]$$

$$\text{or } F_{j+1/2} = \frac{1}{2} [f(u_L) + f(u_R)] - \frac{1}{2} \int_L^R |A(u)| du$$

where  $L = j$  and  $R = j+1$ .

We carry out the integral to obtain a practical formula for 1D Euler eqns.

$$\frac{\partial U}{\partial t} + \frac{\partial f}{\partial x} = 0 \quad \text{or} \quad \frac{\partial U}{\partial t} + A(U) \frac{\partial U}{\partial x} = 0$$

right eigenvectors  $\vec{r}^{(1)}, \vec{r}^{(2)}, \vec{r}^{(3)}$   
 eigenvalues  $\lambda^{(1)} = u - c, \lambda^{(2)} = u, \lambda^{(3)} = u + c$  ( $c^2 = \partial f / \partial p$ )

Char. variables  $v = [v^{(1)}, v^{(2)}, v^{(3)}]^T$

Cons. variables  $U = [u^{(1)}, u^{(2)}, u^{(3)}]^T$

We have  $df = A dU$  (but  $df^\pm \neq A^\pm dU$ ), and  $dU = R dV$

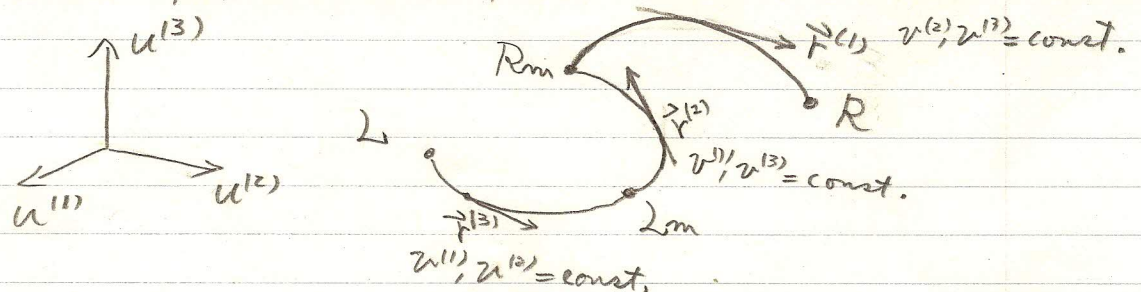
$$\rightarrow df = AR dV \quad (R = [\vec{r}^{(1)}, \vec{r}^{(2)}, \vec{r}^{(3)}])$$

$$\rightarrow df = A \vec{r}^{(1)} dv^{(1)} + A \vec{r}^{(2)} dv^{(2)} + A \vec{r}^{(3)} dv^{(3)}$$

$$\rightarrow df = \lambda^{(1)} \vec{r}^{(1)} dv^{(1)} + \lambda^{(2)} \vec{r}^{(2)} dv^{(2)} + \lambda^{(3)} \vec{r}^{(3)} dv^{(3)}$$

### The integration path

The path is split over all simple wave solution as shown below.



So, we split the integral as follows

$$\int_L^R |A(u)| du = \int_L^{Lm} |A(u)| du + \int_{Lm}^{Rm} |A(u)| du + \int_{Rm}^R |A(u)| du$$

