THE CFD NEWS

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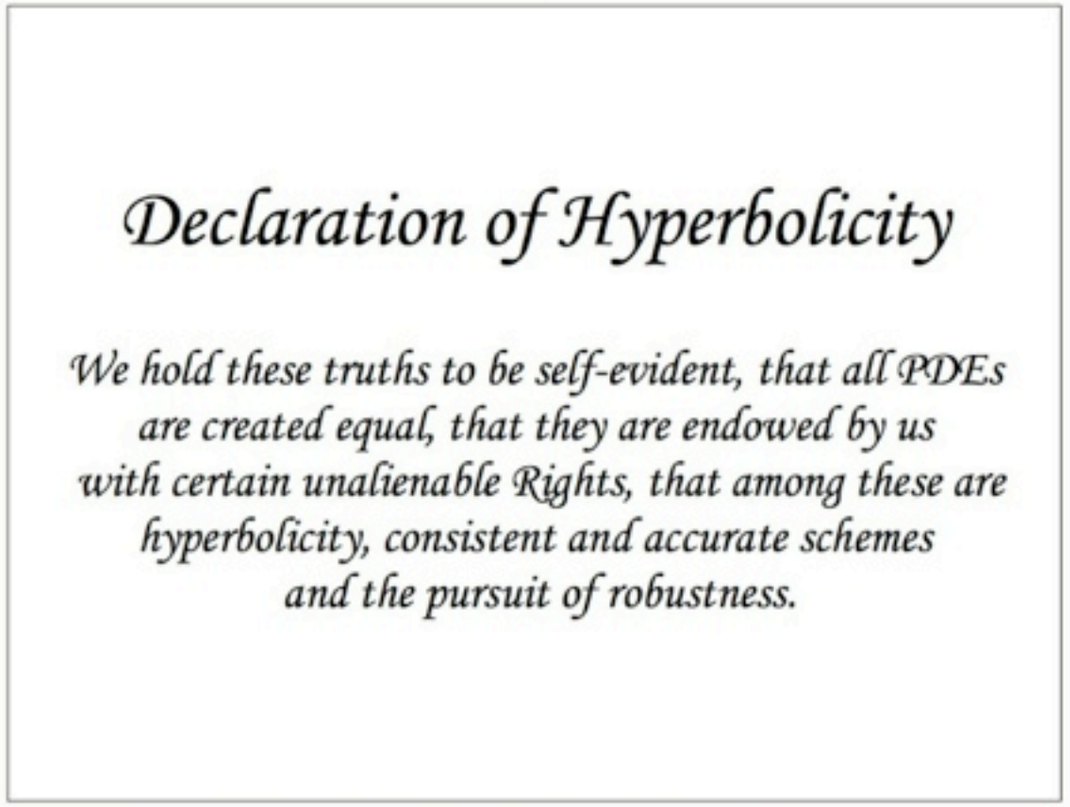
Hyperbolicity Declared for PDEs



Declaration of hyperbolicity presented by Dr. Hiroaki Nishikawa (NIA) at the 51st AIAA Aerospace Sciences Meeting, January 10, 2013, Grapevine, TX, USA

Hyperbolicity was declared, for the first time in the entire CFD history, for all partial differential equations (PDEs) at 51st AIAA Aerospace Sciences Meeting held in Grapevine, Texas, on January 10, 2013. It was declared totally unexpectedly at the technical talk by Dr. Hiroaki Nishikawa, Senior Research Scientist at National Institute of Aerospace (USA). "I was preparing for this moment since 2007. I thought I had to do it now in order to rescue people suffering from various difficulties with non-hyperbolic PDEs like parabolic PDEs," says Nishikawa.

It seems like the declaration came to rescue, but it has generated heated controversy among researchers.

"That's totally crazy. I'm not going to let anyone make me hyperbolic," says Dr. Parabolic at the Viscous Institute of Technology. He claims, "It is simply wrong because each type of PDEs is designed to model specific physical phenomena and one may not change it no matter what. It is a completely wrong idea." Nishikawa argues that the hyperbolicity is just for the sake of numerically solving the PDEs and at the end of the day the numerical solution satisfies the original equations, parabolic or whatever. Parabolic counters,

$$\mathbf{U}_t + \mathbf{A}\mathbf{U}_x = \mathbf{B}\mathbf{U}_{xx} + \mathbf{C}\mathbf{U}_{xxx} + \cdots + \mathbf{S}$$

$$\downarrow$$

$$\tilde{\mathbf{W}}_t + \tilde{\mathbf{A}}\tilde{\mathbf{W}}_x = 0$$

PDE is made hyperbolic by turning non-hyperbolic terms on the right hand side, including a source term, into a hyperbolic system such that it reduces to the original in the steady state.

"Totally insane. Successful numerical schemes should reflect the nature of the PDE they are solving. Upwind scheme for isotropic diffusion has no chance to work." Although it sounds right, the numerical results shown by Nishikawa indicate that the claim is not true. In fact, unusually good results have been obtained by the upwind scheme for diffusion and viscous flow problems. They are unusually good because high-order accurate gradients have been obtained at a dramatically 'reduced' cost.

These interesting results have attracted a number of researchers around the world. Professor Elliptic at the University of Smooth says, "It is quite nice and welcome. As I see it, the hyperbolized parabolic-PDE is hyperbolic in time but remains elliptic in space. It's just like the acoustic sub-system of the Euler equations, which is hyperbolic in time but elliptic in space in subsonic flows." Dr. Muscl at Monotone National Laboratory (currently under reconstruction) is another researcher who welcomed the declaration. He says, "It's a wonderful news. I feel like I've got a lot more places to work at than I thought."

On the other hand, Always Nolimiter, a graduate student of aerospace engineering, says he's been scared to death since he heard the news. He says, "I'm so scared because the hyperbolic Navier-Stokes equations may generate additional shockwaves due to the hyperbolic viscous term. I just don't know what to do. They're gonna blow me up!" According to Professor Elliptic, however, the student is worrying for nothing. He says, "No shockwaves will be generated by the hyperbolic viscous term. Like I said, they are elliptic in space. There will be no shockwaves running across the domain."

Taro Sushiyama, one of the best sushi chefs in town, commented on the analogy of Sushi Burger repeatedly used by Nishikawa to illustrate the concept. He says, "It looks eccentric. It's against tradition and not acceptable in our world. But it's an interesting idea. In another world, maybe, only the taste matters. If Nishikawa-san succeeds, I'll be happy to make a fine sushi burger for him."

While the heated debate continues, progress is being made towards the birth of practical all-hyperbolic CFD codes. The key to success seems to lie in the taste, not in looks, as Sushiyama implied.

Sushi burger, a radical approach. "Looks eccentric, but it's simple to make and tastes the same or even better."