New-Generation CFD Codes by *Hyperbolic* Navier-Stokes System

NATIONAL INSTITUTE OF AEROSPACE

Objective: **Create *new-generation* Computational Fluid Dynamics (CFD) codes that are orders-of-magnitude more efficient and accurate than existing (traditional) codes by revolutionizing the way we solve partial-differential equations numerically.** **"***Turn it into hyperbolic***."**

Approach: First-Order Hyperbolic System Method

Nishikawa, JCP2007
Nishikawa, JCP2010

Diffusion Equation (Parabolic)	First-Order System (Hyperbolic)
$u_t = \nu u_{xx}$	$u_t = \nu p_x$ $p_t = (u_x - p)/T_r$

Two models are equivalent in the steady state for arbitrary relaxation time, Tr.

We compute the steady state solution to the diffusion equation by integrating the hyperbolic system in time to the steady state .

Intrinsic Features:

1. **Methods for hyperbolic systems are directly applicable to diffusion.**
   If you have a good method/textbook for hyperbolic systems, you have a good method/textbook for the diffusion equation. Hard to believe, but now true.

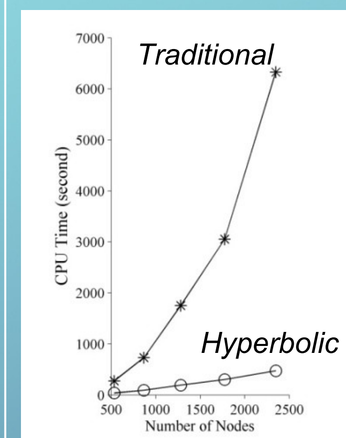
2. **O(1/h) speed-up to obtain the steady state solution.**
   *Time step is not O(h^2) but O(h) because the system is first-order (h is a mesh spacing). The speed-up factor is O(1/h): it gets **faster** for larger-scale problems.*

3. **Higher-order accuracy in the solution gradient, p.**
   The solution gradient is now an additional unknown. It'll be computed to the same order of accuracy as that of the solution (one order less otherwise).

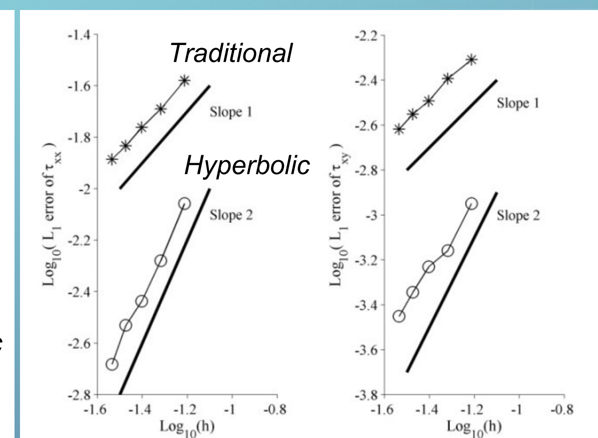
Results:

Nishikawa, AIAA Paper 2011-3043

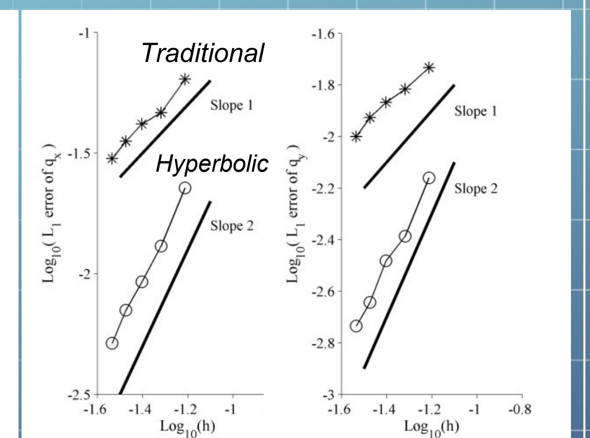
The first-order hyperbolic system method has been extended to the Navier-Stokes system by writing the viscous term as a first-order system with viscous/heat fluxes added as additional unknowns. Left figure shows the CPU time comparison obtained for 2D irregular grid calculations by the hyperbolic method and a traditional method. The hyperbolic method is faster, and gets faster for finer meshes. Right figure shows the error convergence on the viscous/heat fluxes. The hyperbolic method gives 2nd-order accurate viscous/heat fluxes while the traditional method gives only 1st-order accuracy. The results show *orders-of-magnitude* improvements by the hyperbolic method.



- Convergence -



- Error in Viscous stresses -



- Error in Heat fluxes -

We've just opened the door to the next-generation CFD codes. It is only the beginning.

Research Interests:
Algorithm development for Computational Fluid Dynamics.

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