# Every Child Deserves a Bright Future.

I believe it.  I myself have a highly gifted daughter.
But children are children.
Someone needs to take care of them. Usually, a parent.
That's exactly what I've been doing for my daughter.
This is a story about my 5-year old daughter who has an amazing capability for CFD,
and how bright her future can be.
Her name is ....

# First-Order Hyperbolic System Method

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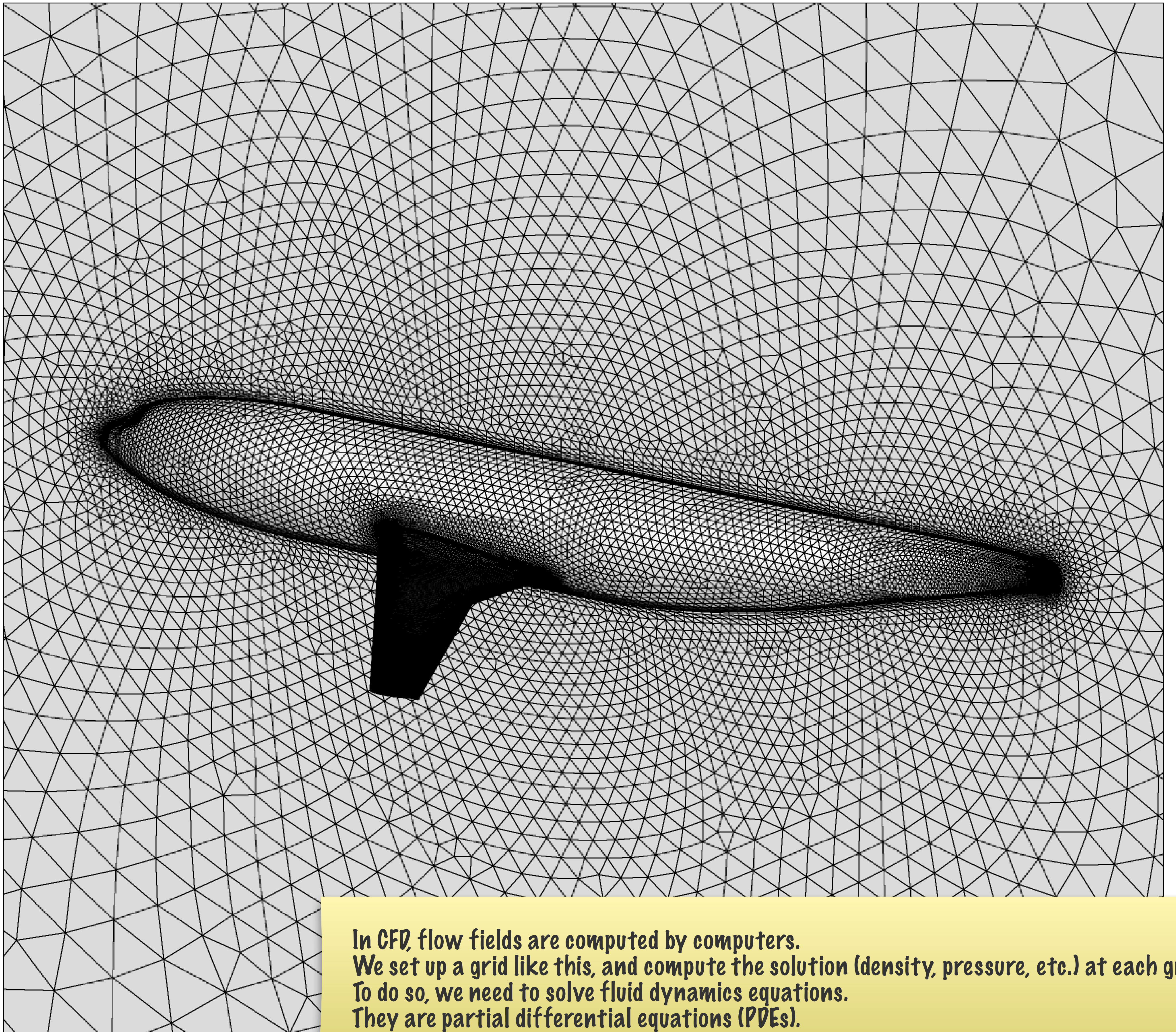


In CFD, flow fields are computed by computers.
We set up a grid like this, and compute the solution (density, pressure, etc.) at each grid point.
To do so, we need to solve fluid dynamics equations.
They are partial differential equations (PDEs).

# Partial Differential Equations

## Physical phenomena modeled by various terms:

$$\mathbf{U}_t + \mathbf{A}\mathbf{U}_x = \mathbf{B}\mathbf{U}_{xx} + \mathbf{C}\mathbf{U}_{xxx} + \cdots + \mathbf{S}$$

| | Hyperbolic | Parabolic | Dispersion | Source |
|---|---|---|---|---|
| *Difficulty* → | | | | |
| **Time step** | $O(h)$ | $O(h^2)$ | $O(h^3)$ | |
| **Algorithm** | Well developed | No so well | Not so well | Tricky |

For implicit schemes, this is related to the conditioning of residual Jacobian.

Hyperbolic term used to be very difficult (shock waves), but a great progress was made in the past decades, and methods are well understood.
Many textbooks are now available. The current focus of algorithm development is the parabolic term (high-order, unstructured).
It looks like the development for numerical methods continues for another decade or two...
But here she comes and says "If it is easy for the hyperbolic term, and difficult for others. Why don't we just....".

# Turn It into Hyperbolic

## First-Order Hyperbolic System Method - JCP2007

$$\mathbf{U}_t + \mathbf{A}\mathbf{U}_x = \mathbf{B}\mathbf{U}_{xx} + \mathbf{C}\mathbf{U}_{xxx} + \cdots + \mathbf{S}$$

$$\Downarrow$$

$$\tilde{\mathbf{W}}_t + \tilde{\mathbf{A}}\tilde{\mathbf{W}}_x = 0$$

Dramatic simplification/improvements to numerical methods.

1. Method for hyperbolic systems applies to all.
2. Tremendous speed-up in computation: O(h) time step.
3. Exceptionally high-accurate derivative quantities.

It is DRAMATIC. All high-order derivatives as well as source terms are transformed into a hyperbolic system.
The idea was originally proposed for the second derivative (JCP2007), but it is actually extendible to all derivatives and source terms.
If you don't really understand what it means, this is what it means: Go to the next.

# Turn Every Food into a Burger!

*Simple, Efficient, Accurate.*

**Sushi Burger!**


Picture borrowed from a blog talking about an existing sushi burger shop in AU. Thank you.

If it is difficult for you to make sushi, make it a sushi burger.
It is simple, efficient, and accurate.
No formal training is required. You cannot make mistakes in making sushi burgers!
Just make it simple like a burger.

I know it is a crazy idea.
I don't actually know as a Japanese if I like sushi burgers...
But this is what it means to turn a non-hyperbolic term into hyperbolic.
If you don't like the idea, it may just be due to a cultural shock or a psychological barrier.

# Hyperbolic Advection-Diffusion System

( JCP 2010 )

## Advection-Diffusion Equation

$$u_t + u_x = u_{xx}$$

Similar to mixed finite-element, FOSLS, LSFEM? Yes, similar, but different in that the system here is HYPERBOLIC while their systems have no such characterization. It is the hyperbolicity that brings the advantages.

## *Hyperbolic Advection-Diffusion System*

$$u_t + u_x = p_x$$

$$p_t = u_x - p$$

$$\tilde{\mathbf{W}}_t + \tilde{\mathbf{A}}\tilde{\mathbf{W}}_x = \tilde{\mathbf{S}}$$

$$\tilde{\mathbf{W}} = [u, p]$$

Two models are equivalent in the steady state.
Same accuracy for all variables, including p.

Hyperbolic system is constructed by adding an extra variable and its equation.
It is deliberately constructed such that it reduces to the original equation in the steady state.
A very efficient and accurate steady solver is then generated.
It can be used as a building block for time-accurate simulation (dual-time approach).
The same order of accuracy is achieved for both u and p; p will be the solution gradient in the steady state.

# Hyperbolic Navier-Stokes System

( AIAA2011-3043 )

Navier-Stokes System (hyperbolic + parabolic)

$$\mathbf{U}_t + \mathbf{A}\mathbf{U}_x = \mathbf{B}\mathbf{U}_{xx} \qquad \mathbf{U} = [\rho, v, p]$$

*Hyperbolic Navier-Stokes System (hyperbolic)*

$$\tilde{\mathbf{W}}_t + \tilde{\mathbf{A}}\tilde{\mathbf{W}}_x = 0 \qquad \tilde{\mathbf{W}} = [\rho, v, p, \tau, q]$$

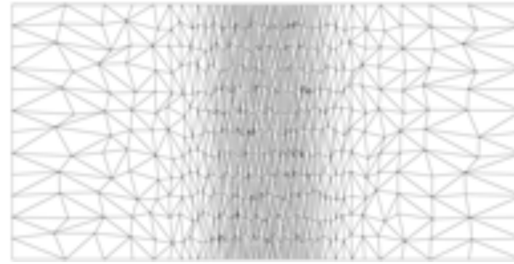
Same can be done for general nonlinear PDEs.

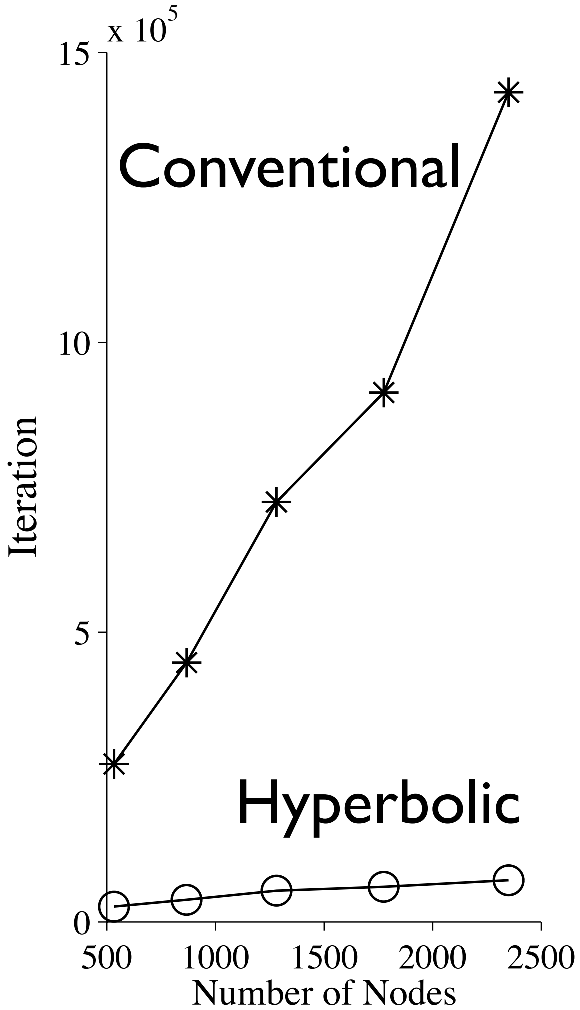
The idea was extended to the compressible Navier-Stokes equations in 2011.
The hyperbolic NS system was constructed by adding the viscous stresses and the heat fluxes as new unknowns.
Just by solving the hyperbolic system instead of the original system, you enjoy all those advantages in any numerical method.
Yes, it is independent of numerical methods. Advantages come from the alternative form of the PDE.

# Results are Amazing

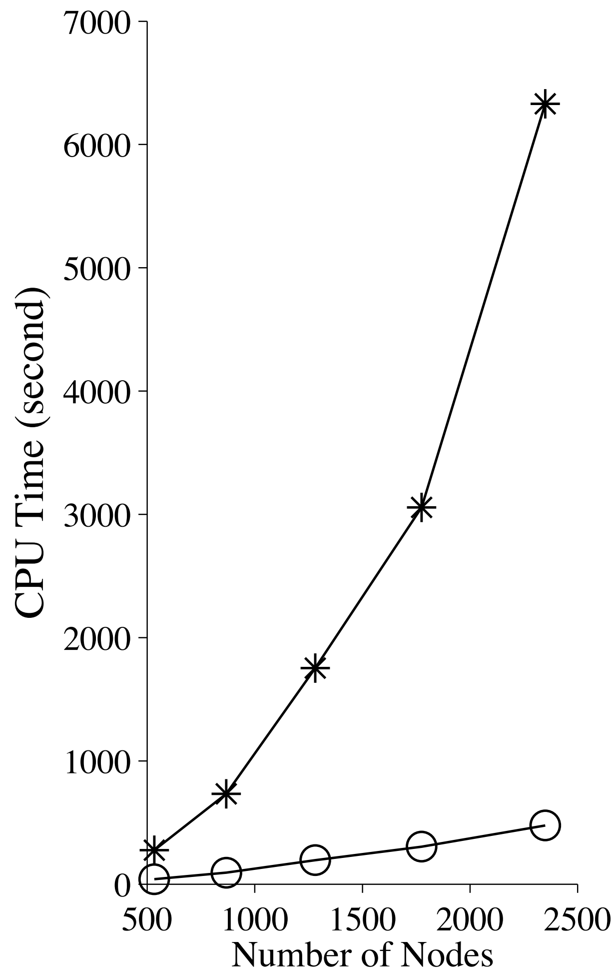
## Viscous Shock-Structure Problem

### 2nd-order finite-volume method

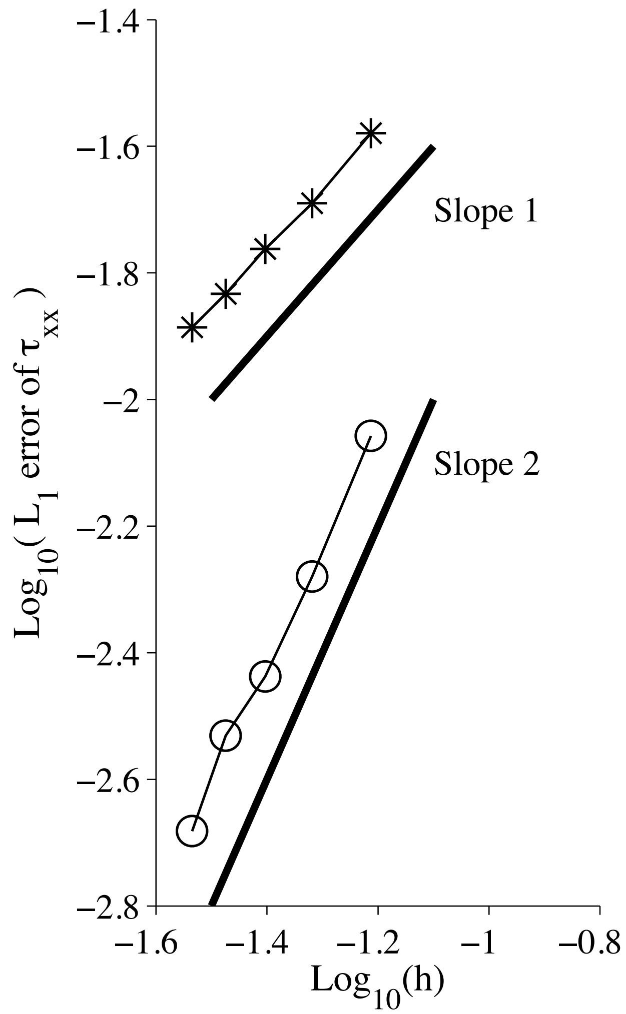




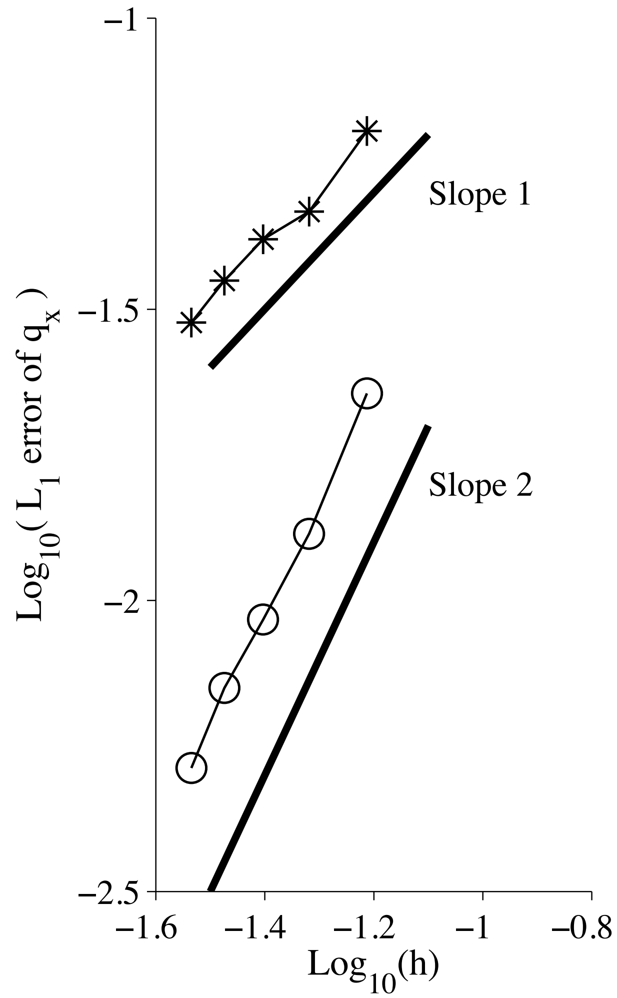

Iteration




CPU Time




Viscous Stress




Heat Flux

Steady solution was computed by explicit schemes based on original NS and hyperbolic NS systems.
The speed-up factor is O(1/h), which grows for larger-scale problems.
Second-order accuracy for viscous and heat fluxes is achieved just by solving the hyperbolic system by a second-order scheme.

# Future is Bright

Hitherto unexpected computational advantages brought into numerical methods for PDEs.

$$\tilde{\mathbf{W}}_t + \tilde{\mathbf{A}}\tilde{\mathbf{W}}_x = 0$$

*She'll be the mother of future numerical methods.*

A good method for hyperbolic terms immediately turns into a very good method for all other terms in PDEs.
Advantages will be shared with everyone in all fields where PDEs are solved numerically.
Her future is bright.
On the other hand, my future depends on funding.
No matter what happens, until the day comes when she stands on her own, I'll continue to take care of her.
It is my responsibility as her parent.