Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

A Consistent Hybrid LES/RANS Framework with High-Order LES Solver on Cartesian Mesh

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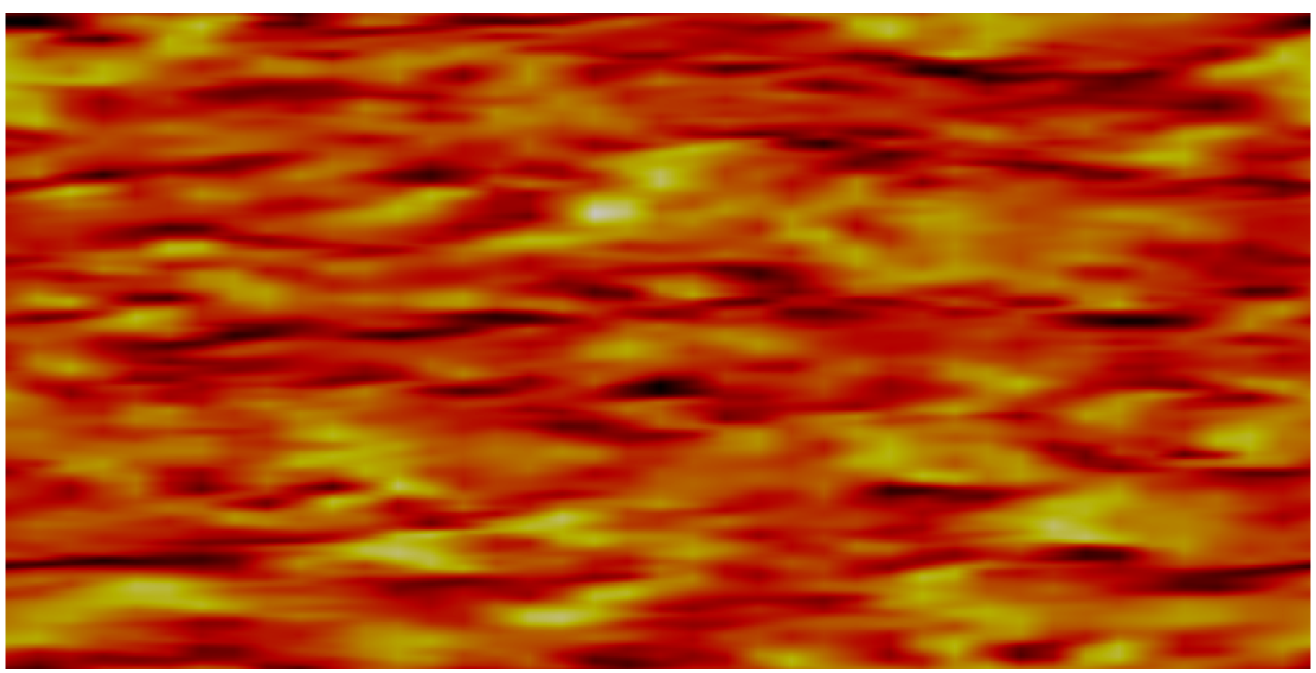
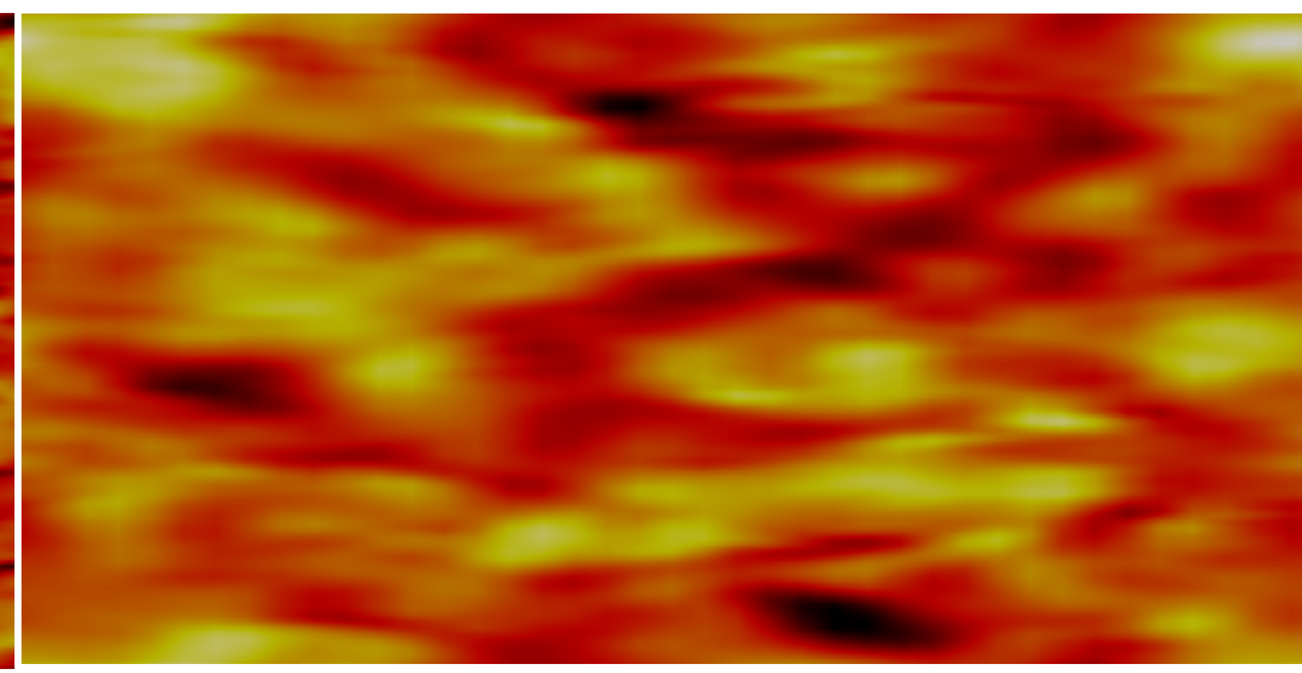
Large Eddy Simulation for Wall-bounded Flows

- Large Eddy Simulation: only resolve large/important scales; model small scales; very successful for free-shear flows.
- What about wall-bounded flows? The small scales (near-wall eddies) are also important.





Near Wall u Channel Center

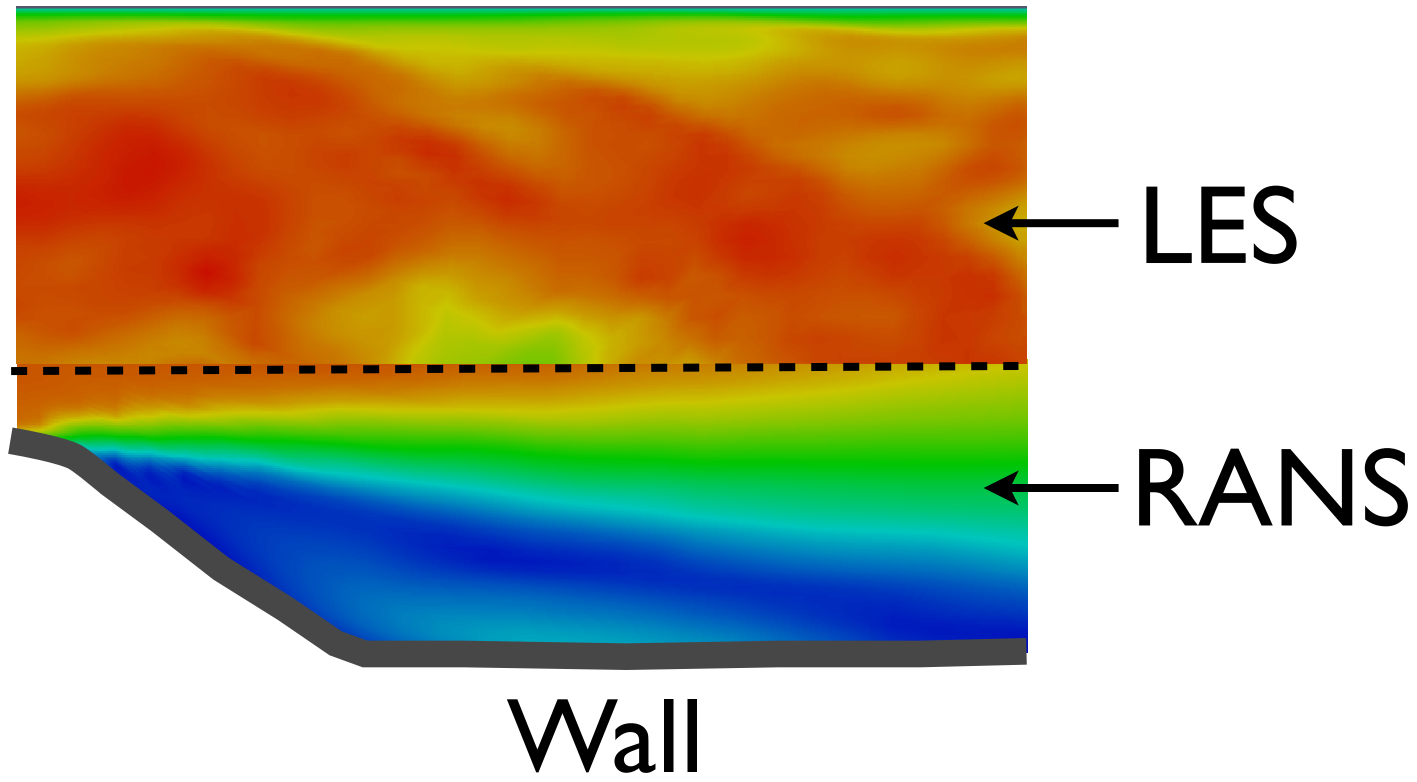
- LES loses its advantage! LES: $O(Re^{1.8})$ *v.s.* DNS: $O(Re^{9/4})$ (Chapman 1974)

Hybrid LES/RANS Methods:
Zonal Approach

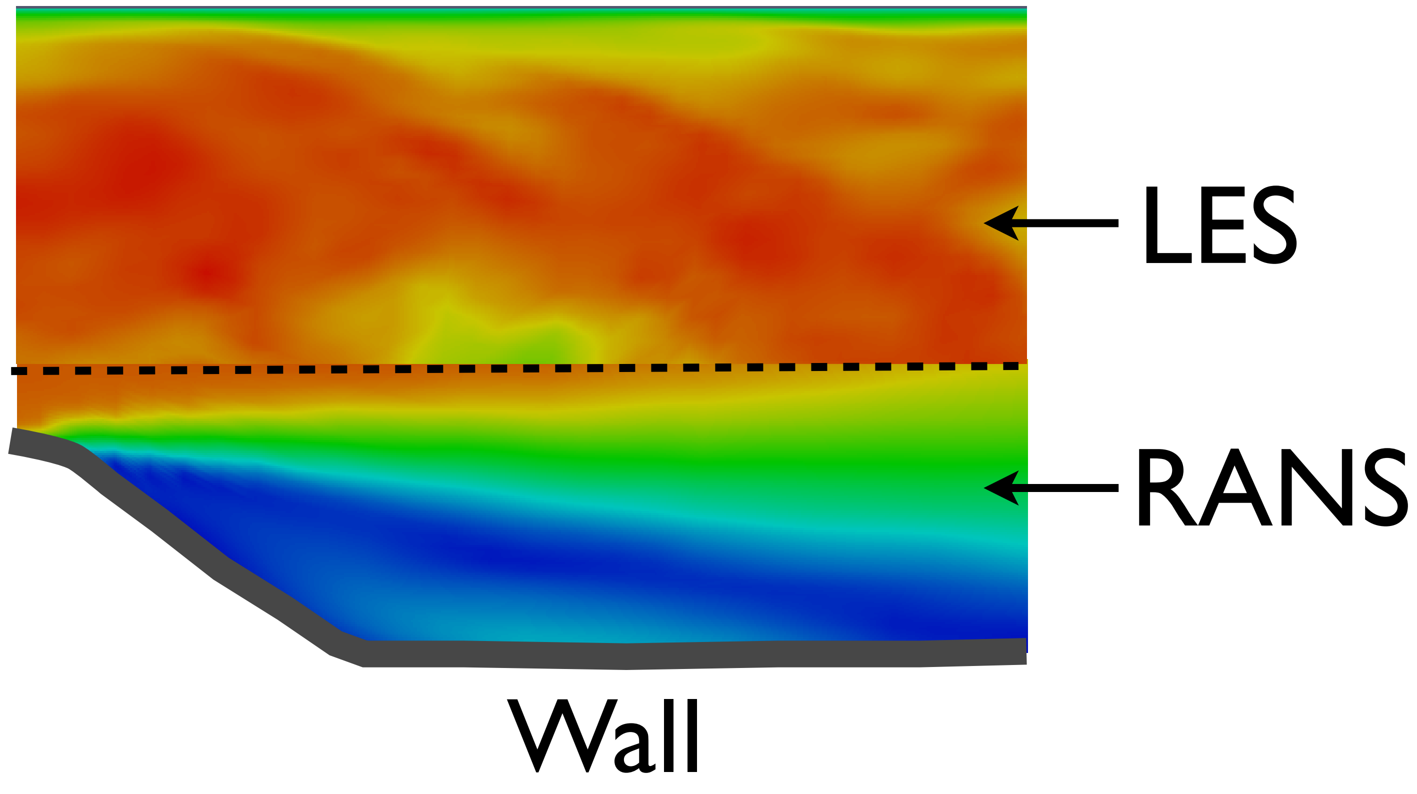
- DES: Fundamental inconsistencies: filtered vs. Reynolds-averaged quantities.
- The inconsistency at the LES/RANS interface leads to modeled stress depletion and log-layer mismatch

Hybrid LES/RANS Method: Consistent Approach

- To overcome this difficulty, we propose a dual-mesh consistent framework for hybrid LES/RANS modeling.



Hybrid LES/RANS Method: Consistent Approach



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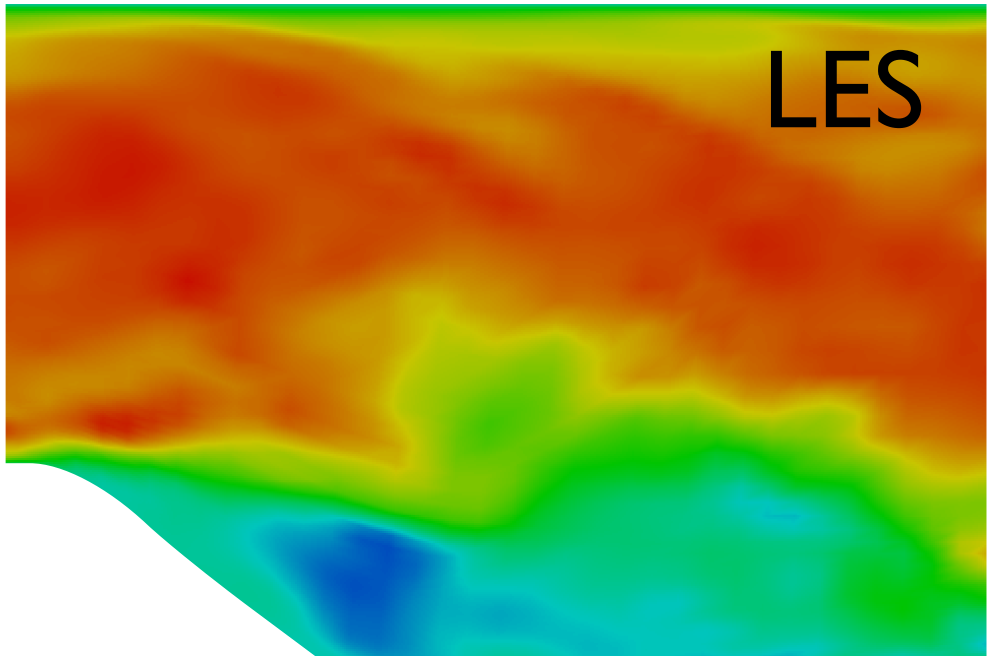

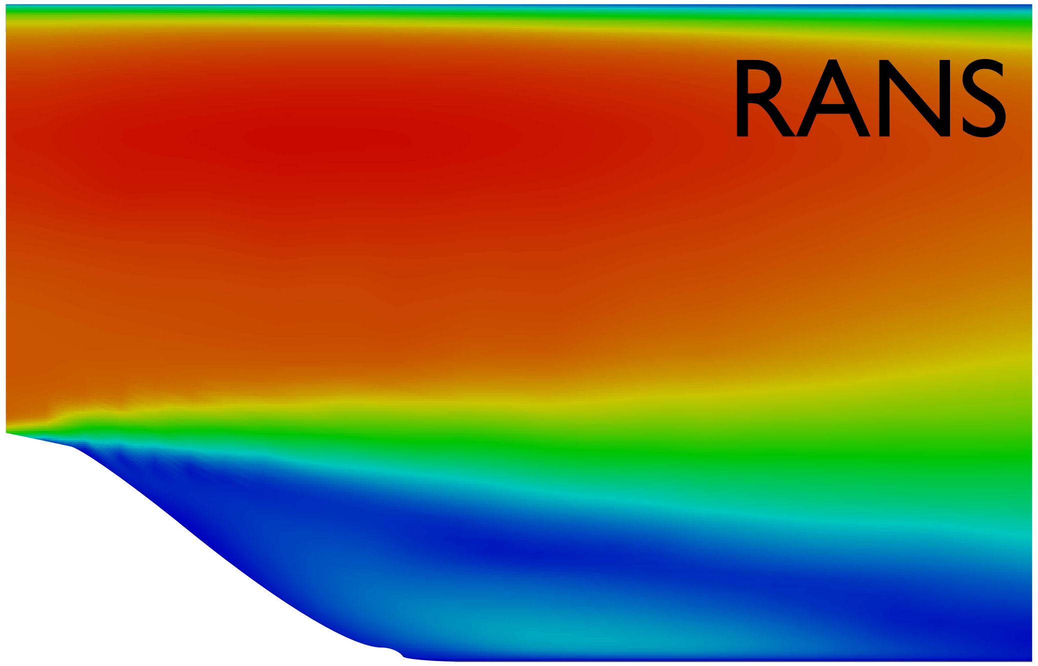

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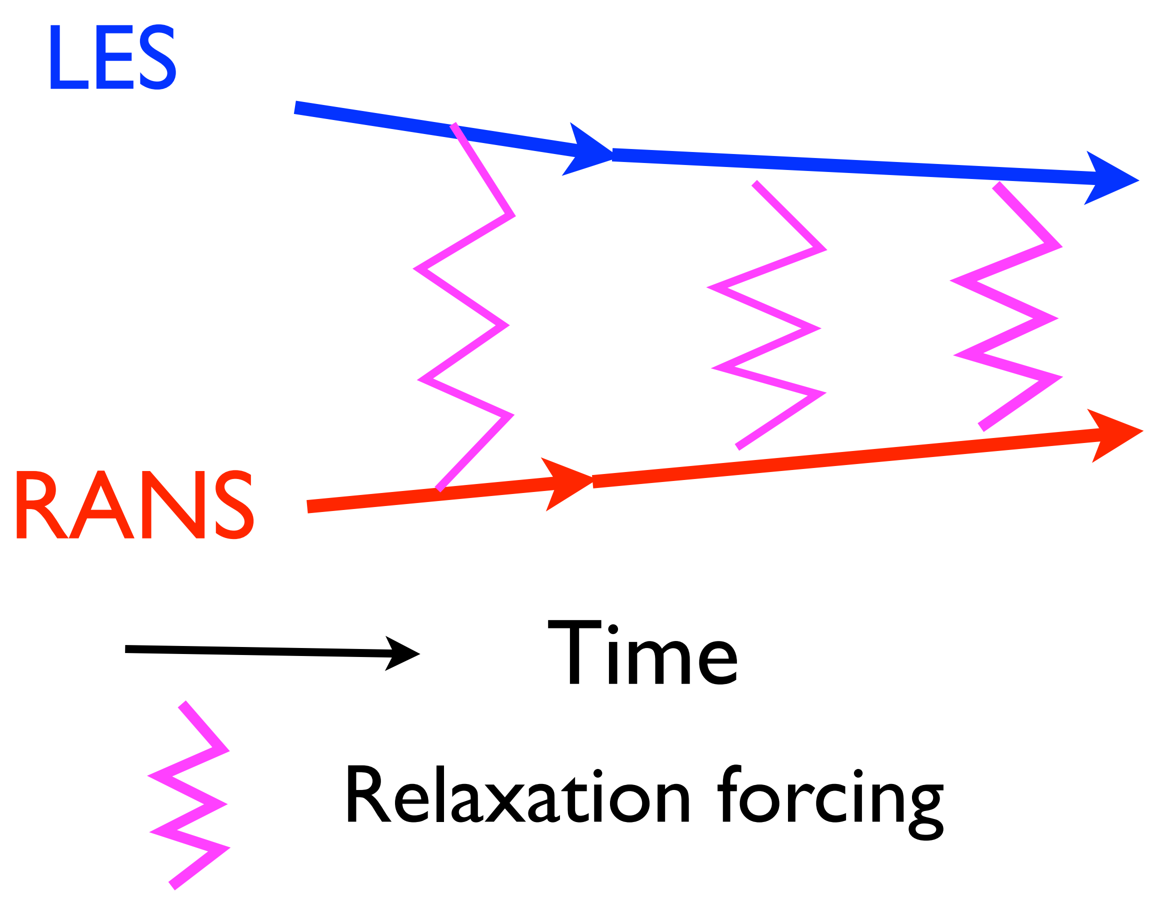

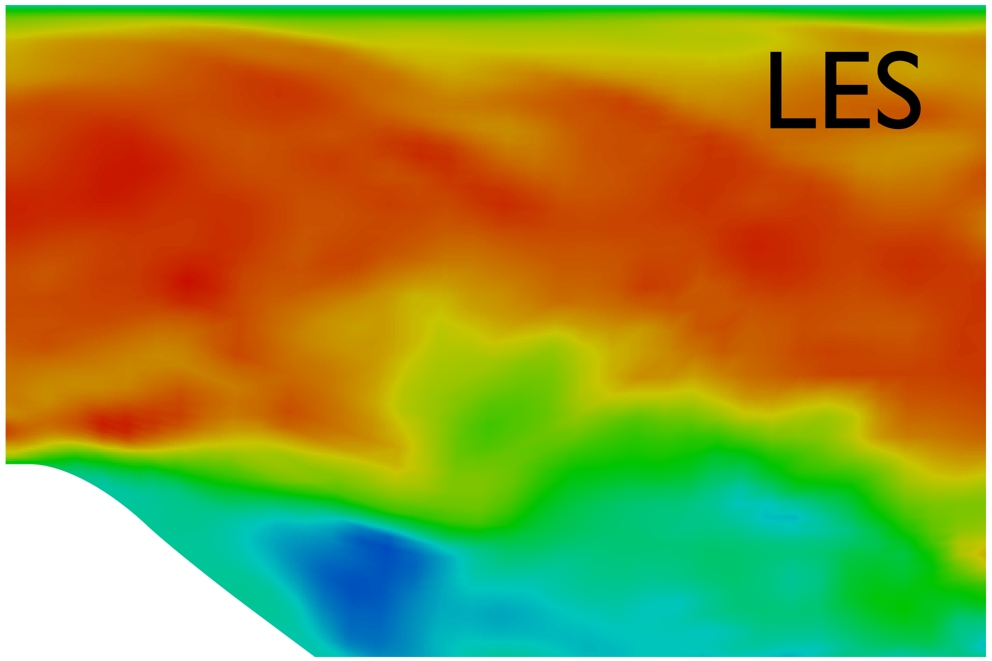

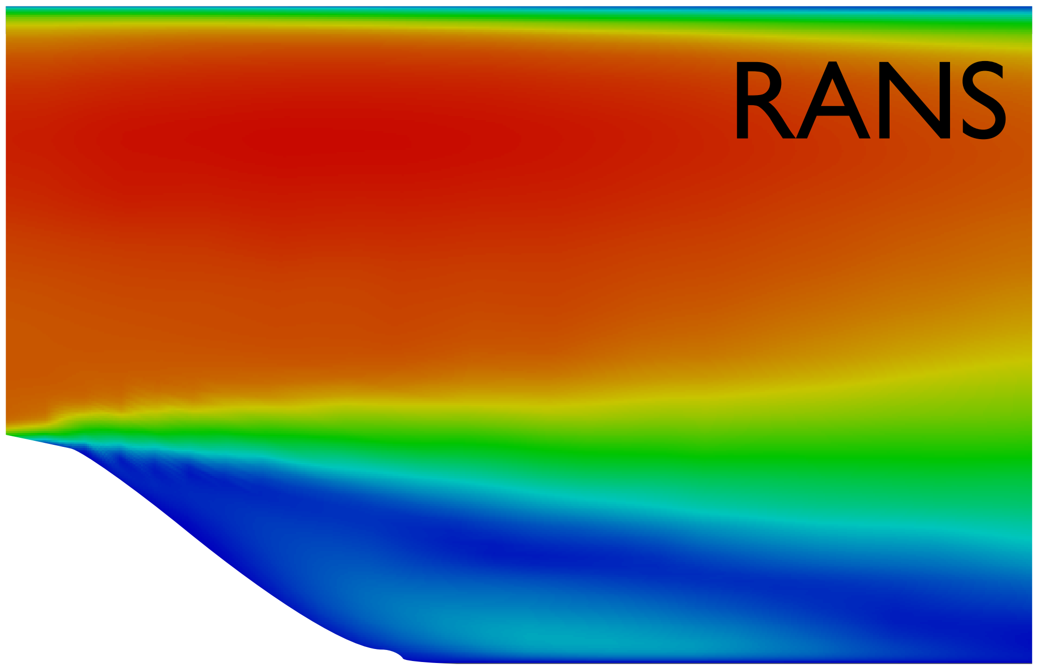

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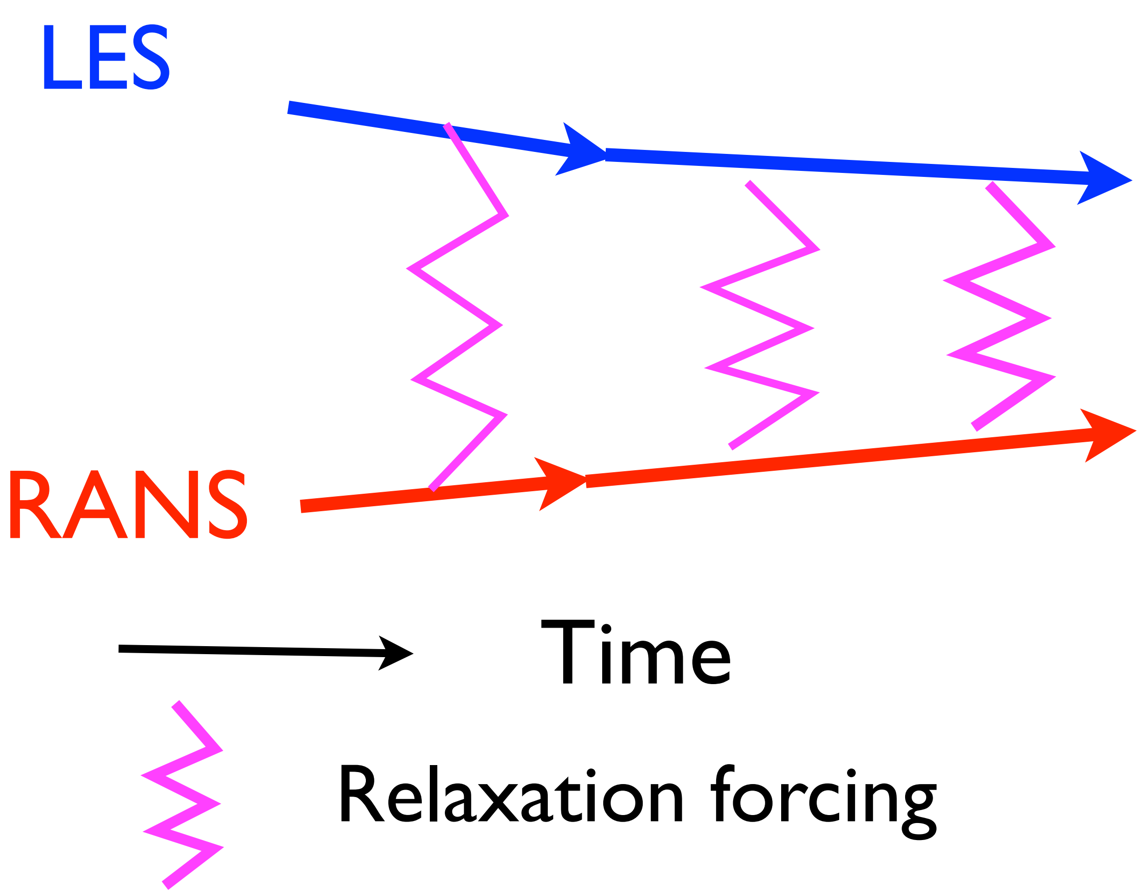

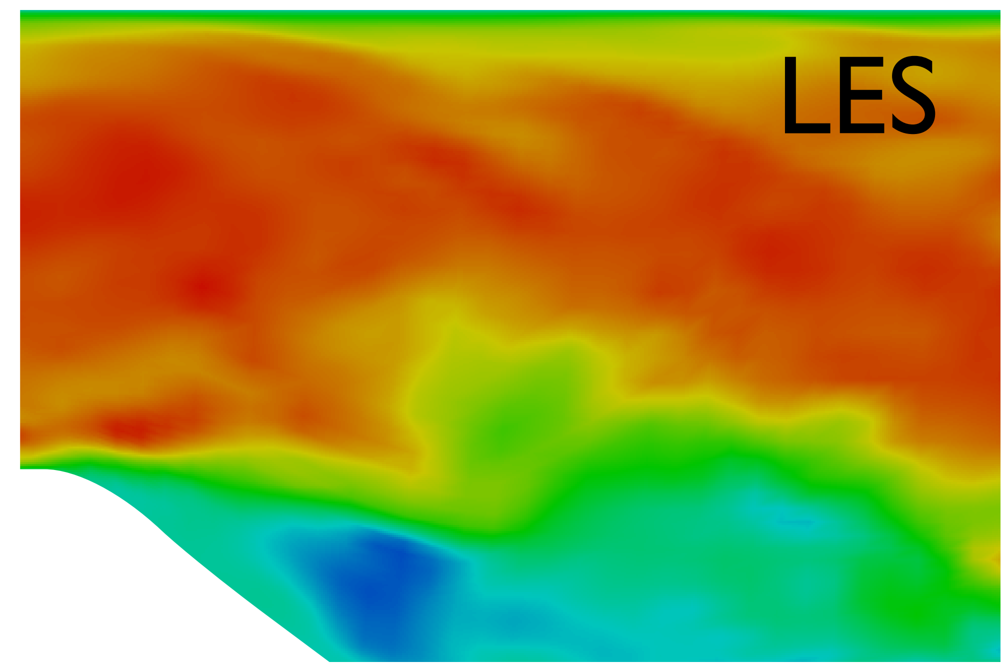

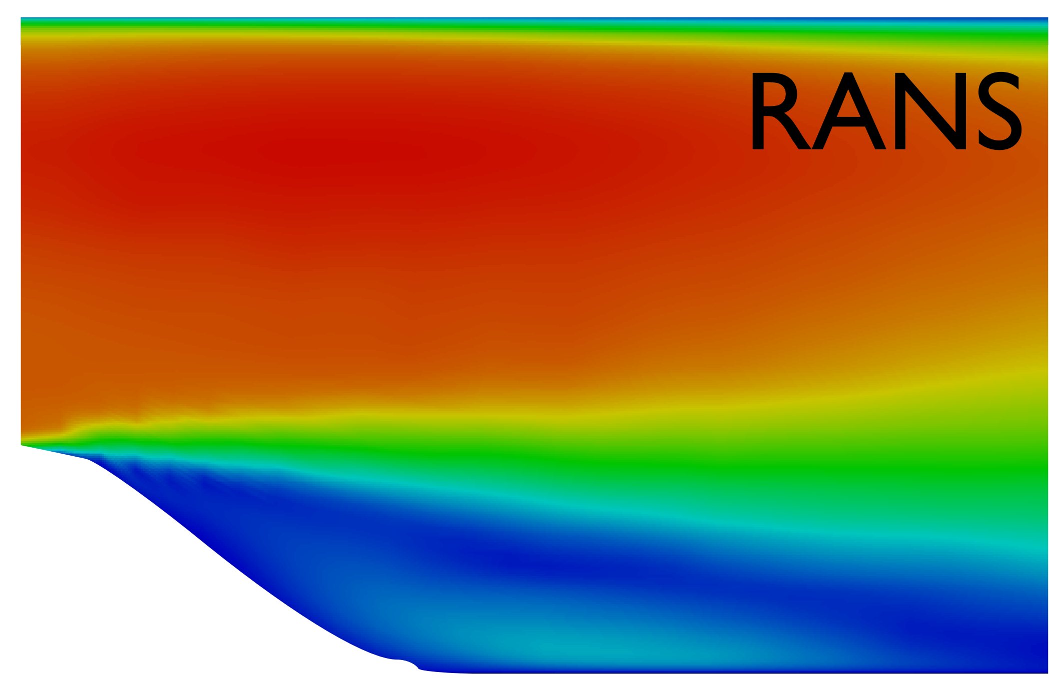

Consistency: the average of LES quantity should be approximately equal to RANS quantity

Hybrid LES/RANS Framework: Forcing

$$\frac{\partial}{\partial t}U_i^* + \frac{\partial}{\partial x_j}\left(U_i^* U_j^*\right) = -\frac{1}{\rho}\frac{\partial}{\partial x_i}p^* + \nu\frac{\partial^2}{\partial x_j \partial x_j}U_i^* - \frac{\partial}{\partial x_j}\tau_{ij}^* + Q_i^*$$

$$\frac{\partial^2}{\partial x_i \partial x_i}p^* = -\rho\frac{\partial^2}{\partial x_i \partial x_j}\left(U_i^* U_j^* + \tau_{ij}^*\right) + \rho\frac{\partial}{\partial x_i}Q_i^*$$

RANS

- $U_i^* = \langle U_i \rangle$ (Reynolds averaged velocity)
- $p^* = \langle p \rangle$ (Reynolds averaged pressure)
- $\tau_{ij}^* = \langle u_i u_j \rangle$ (Reynolds stress)

LES

- $U_i^* = \bar{U}_i$ (filtered velocity)
- $p^* = \bar{p}$ (filtered pressure)
- $\tau_{ij}^* = \tau_{ij}^R$ (residual stress)

Operators and Consistency

- Spatial filtering operator (LES): $\bar{f}$
- Reynolds averaging operator: $\langle f \rangle$
- Exponentially-weighted time average:

$$\langle f \rangle^{\mathrm{AVG}} = \frac{1}{T} \int_0^\infty f(t - t') e^{-t'/T} \, dt'$$

$$\langle U_i \rangle^{\mathrm{LES}} = \langle \bar{U}_i \rangle^{\mathrm{AVG}}$$

$$\langle u_i u_j \rangle^{\mathrm{LES}} = \frac{1}{T} \int_0^\infty [(\bar{U}_i - \langle \bar{U}_i \rangle^{\mathrm{LES}})(\bar{U}_j - \langle \bar{U}_j \rangle^{\mathrm{LES}}) + \tau_{ij}^{\mathrm{R}}] \, e^{-t'/T} \, dt'$$

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resolved stress

residual stress

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Consistency:

$$\langle U_i \rangle^{\mathrm{LES}} \approx \langle U_i \rangle$$

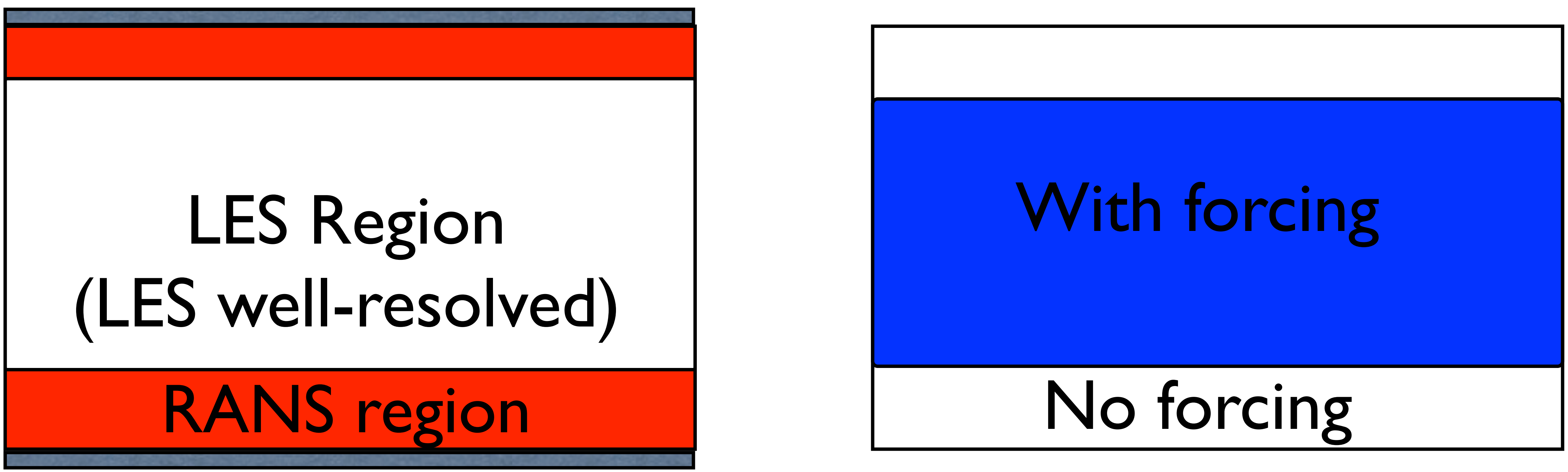
$$\langle u_i u_j \rangle^{\mathrm{LES}} \approx \langle u_i u_j \rangle$$

Hybrid LES/RANS Framework: Forcing

- Forcing on RANS Equations:

$$Q_i^* = \langle Q_i \rangle = (\langle U_i \rangle^{\mathrm{AVG}} - \langle U \rangle_i)/\tau$$

Only active in LES region (well-resolved by LES)!



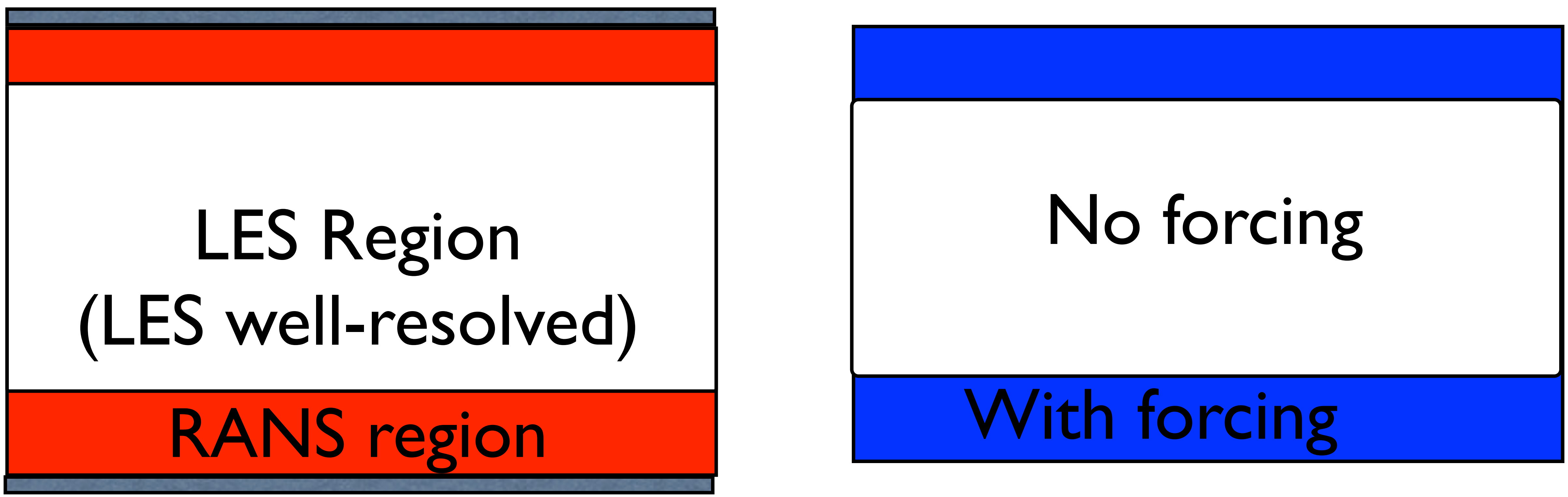
Hybrid LES/RANS Framework: Forcing

- Forcing on LES Equations:

  In RANS region (under-resolved by LES) only:

$$Q_i^* = \bar{Q}_i = (\langle U_i \rangle - \langle U_i \rangle^{\mathrm{AVG}})/\tau_1 + G_{ij}(\langle U_j \rangle^{\mathrm{AVG}} - \bar{U}_j)/\tau_2$$

with $$G_{ij} = \left(\langle u_i u_j \rangle^{\mathrm{AVG}} - \langle u_i u_i \rangle \right) / (u_k u_k)$$



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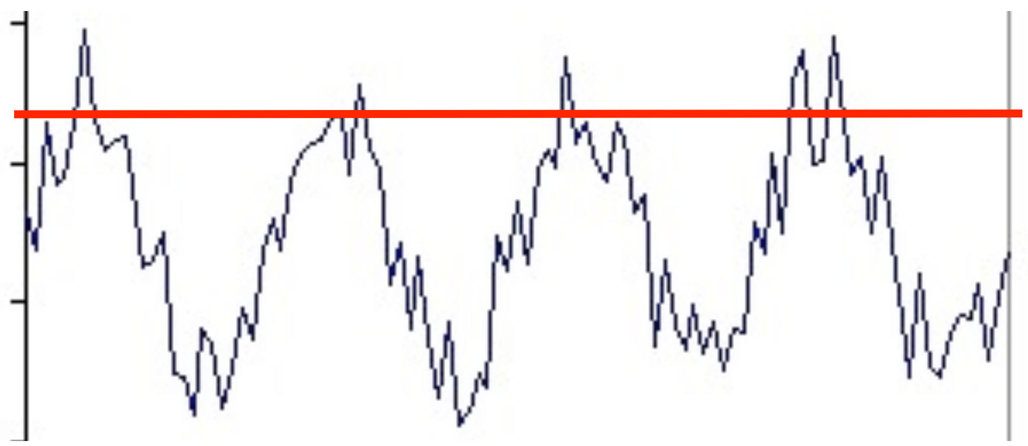
Shift mean without affecting the fluctuations



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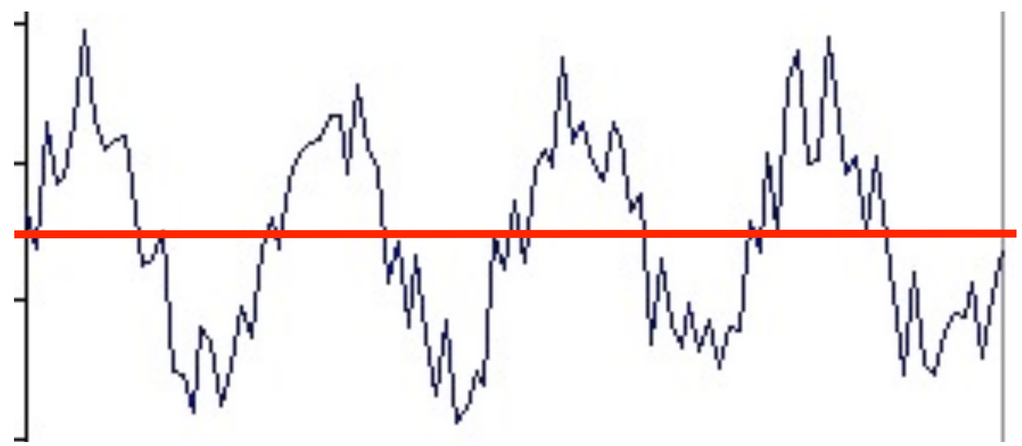

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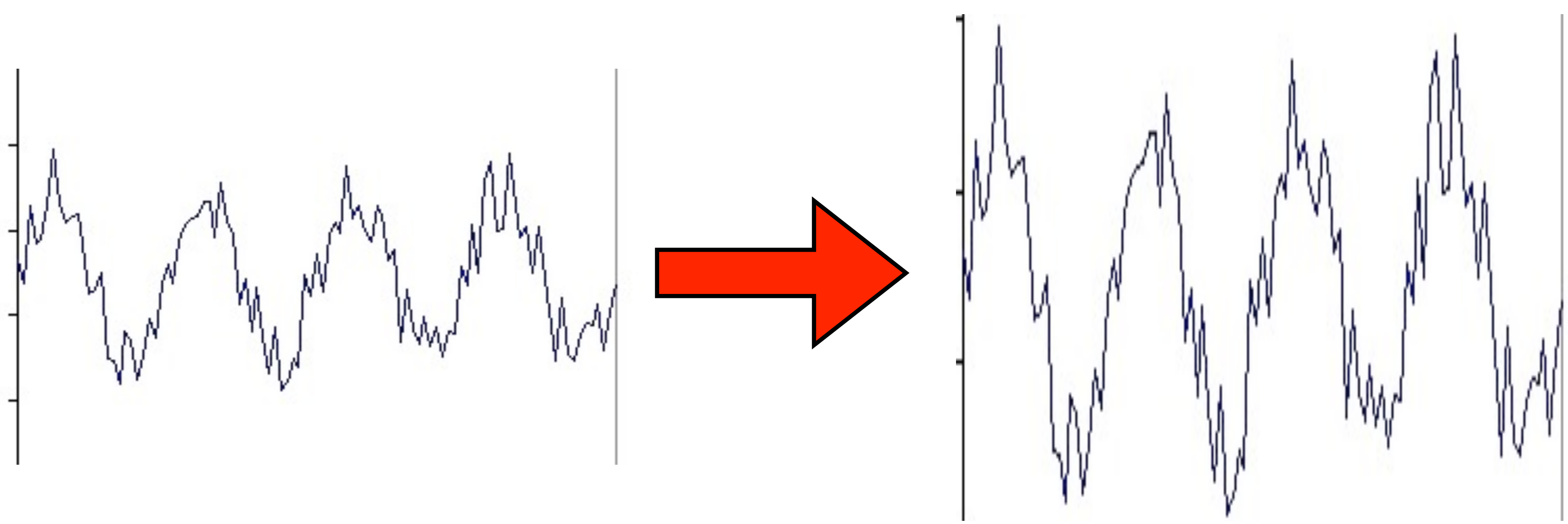
Scale fluctuations without affecting the mean



LES/RANS Algorithm

1. Choose LES and RANS models and other parameters;
2. Initialize the fields for LES and RANS;
3. Initialize the fields for Exponentially Weighted Average (EWA) quantities;
4. Initialize all drift terms to zeros;
for *each time step* **do**
  1. For each grid cell determine whether it is well resolved (in LES), and assign it to LES or RANS region accordingly;
  2. Solve filtered momentum and pressure Poisson equations (2) for $\overline{U}_i$ and $\bar{p}$;
  3. Solve averaged momentum and pressure Poisson equations (3) for $\langle U_i \rangle$ and $\langle p \rangle$;
  4. Update EWA quantities $\langle \overline{U}_i \rangle^{\mathrm{AVG}}, \langle \tau_{ij} \rangle^{\mathrm{AVG}}$ and $\langle \varepsilon \rangle^{\mathrm{AVG}}$, according to Eq. (5);
  5. Interpolate the quantities needed for the calculation of drift terms;
  6. Update drift terms according to Eqs. (10), (15), and (16);
end

Dual-Mesh LES/RANS

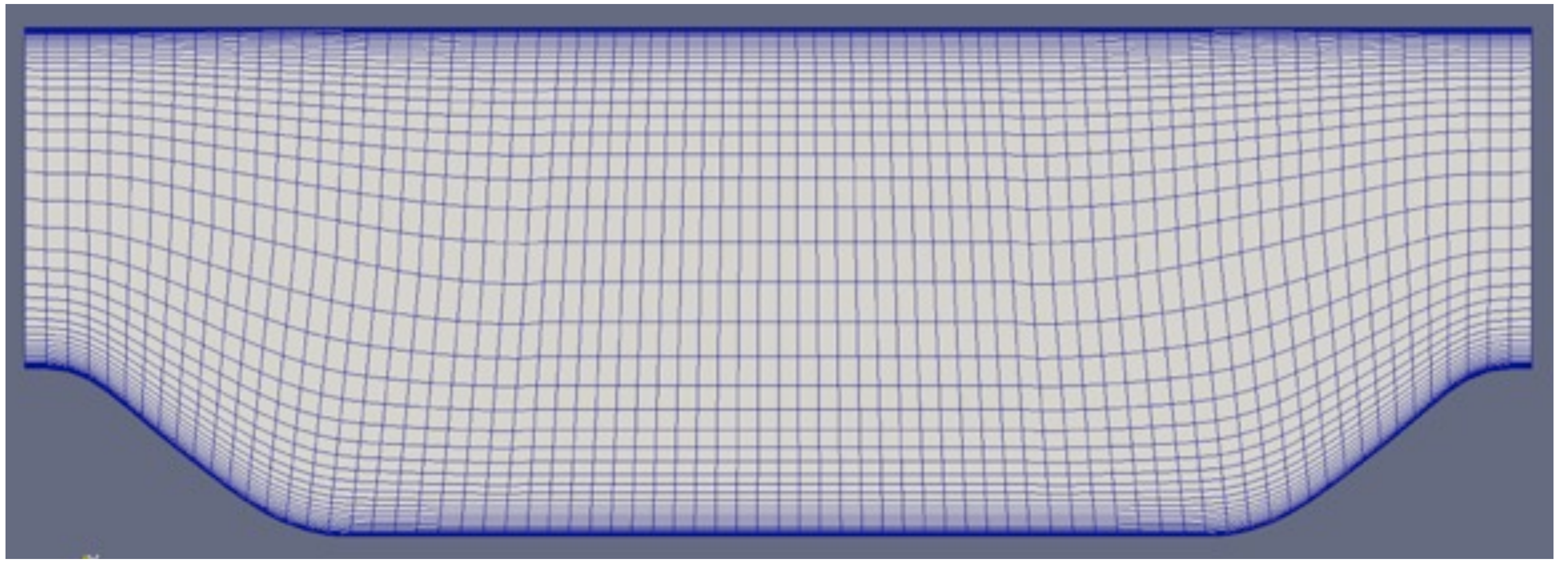
- Filtered and RANS equations are solved simultaneously on separate meshes.
- Interpolations are required if the meshes are not identical.
- Flexibility of using preferred LES and RANS code with minor modification: minimally intrusive.

Possible to use almost any combination of LES and RANS solvers!

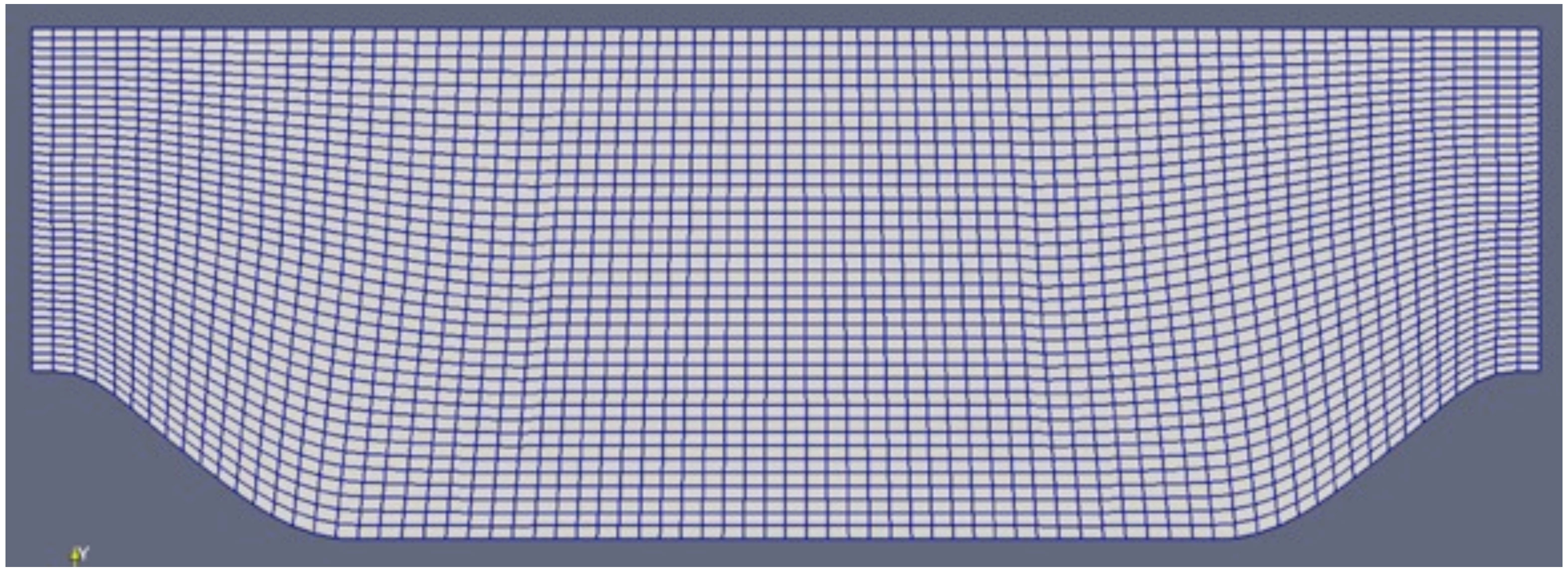
Dual-Mesh Hybrid LES/RANS

- RANS grid optimized for wall bounded flow; LES grid optimized for free-shear flow.
- The LES and RANS solvers only exchange quantities to compute relaxation forces.

RANS:



LES:



Proof-of-Concept Implementation

- We have implemented the dual-mesh LES/RANS framework based on OpenFOAM (standard open-source CFD platform www.openfoam.org)
- Demonstrated the merits on canonical flows
- Both RANS and LES equations are discretized by using 2nd scheme on unstructured meshes.

H. Xiao and P. Jenny (2012). A Consistent dual-mesh framework for hybrid LES–RANS modeling. *Journal of Computational Physics.* 231(4), 1848–1865.

H. Xiao, Y. Sakai, R. Henniger, and P. Jenny. Simulating Flow over Periodic Hills Using a Dual-Mesh Hybrid Solver with High-Order LES, ICCFD7-1604, *Seventh International Conference on Computational Fluid Dynamics (ICCFD7), Big Island, Hawaii, July 9-13, 2012*

http://www.iccfd.org/iccfd7/assets/pdf/papers/ICCFD7-1604_paper.pdf

Using High-Order LES: Motivation

- In LES conducted using second-order schemes, truncation error is $O(h^2)$; which is of the same order as SGS model term e.g., $\nu_{\mathrm{sgs}} = (C_s \Delta)^2 |S_{ij}$
- LES are often computationally expensive. General-purpose CFD codes are designed for flexibility, not for efficiency!

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Strategy: high-order LES + low-order RANS

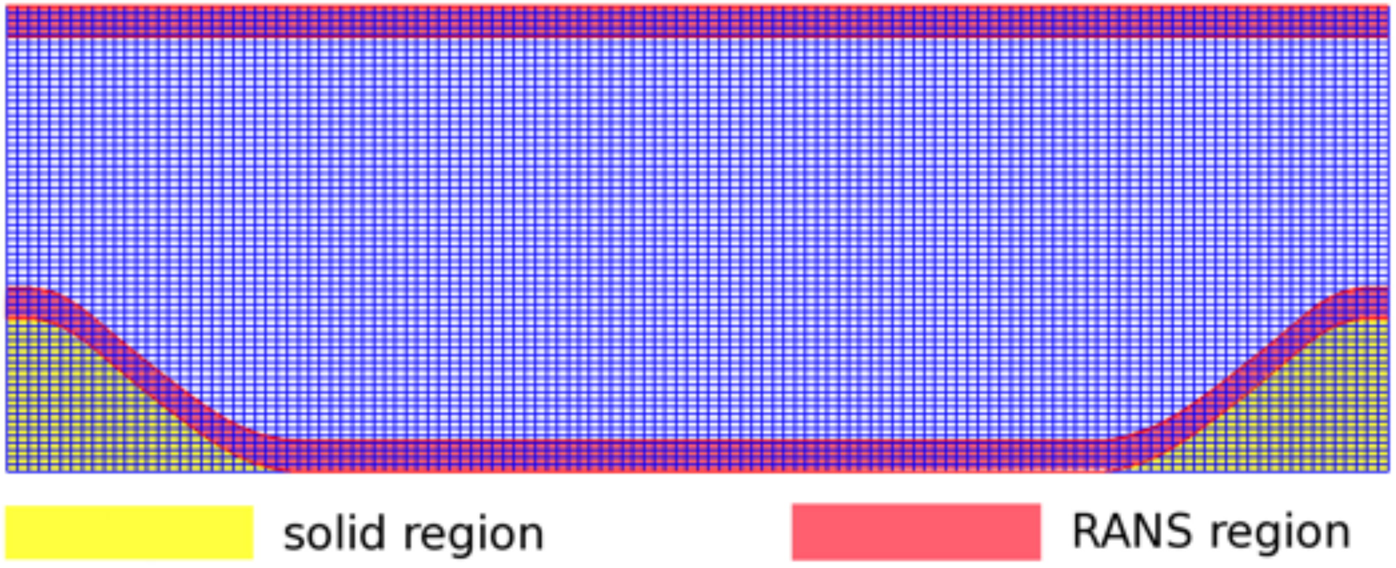
IMPACT OpenFOAM

(Henniger et al. 2012, JCP)

IMPACT/OpenFOAM Coupling: Meshes

- High-accuracy LES on Cartesian grid boundary conditions imposed via immersed boundary technique.

LES on Cartesian grid



IMPACT/OpenFOAM Coupling: Meshes

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- Low-order RANS solver on unstructured grid.

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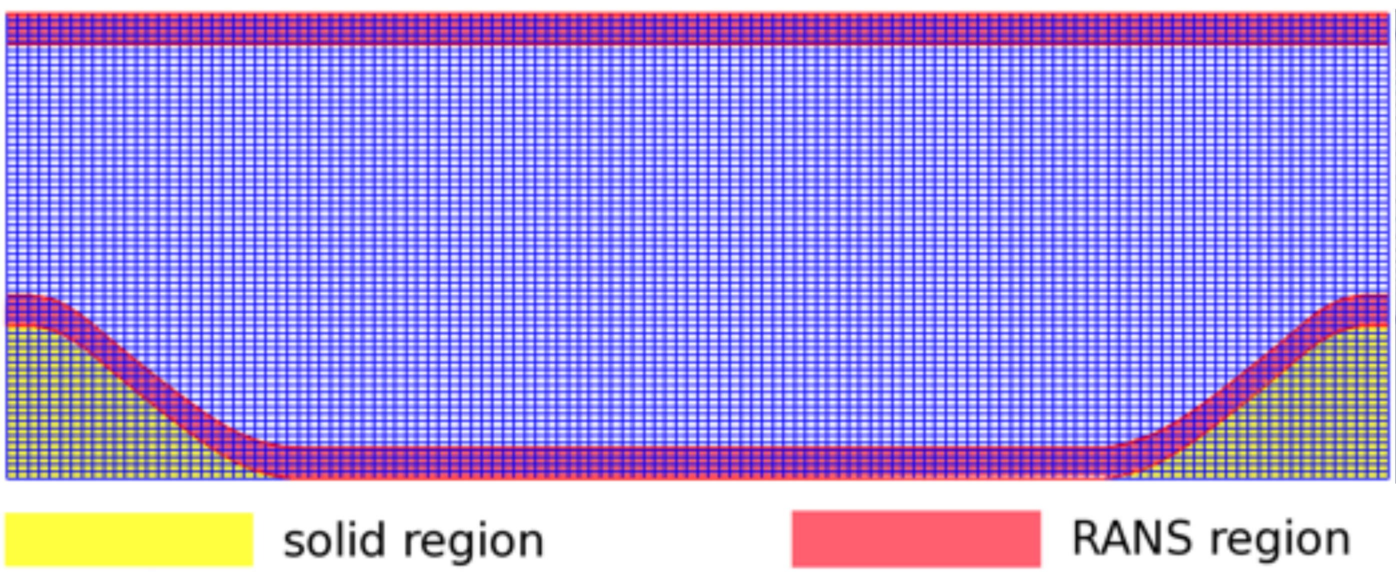
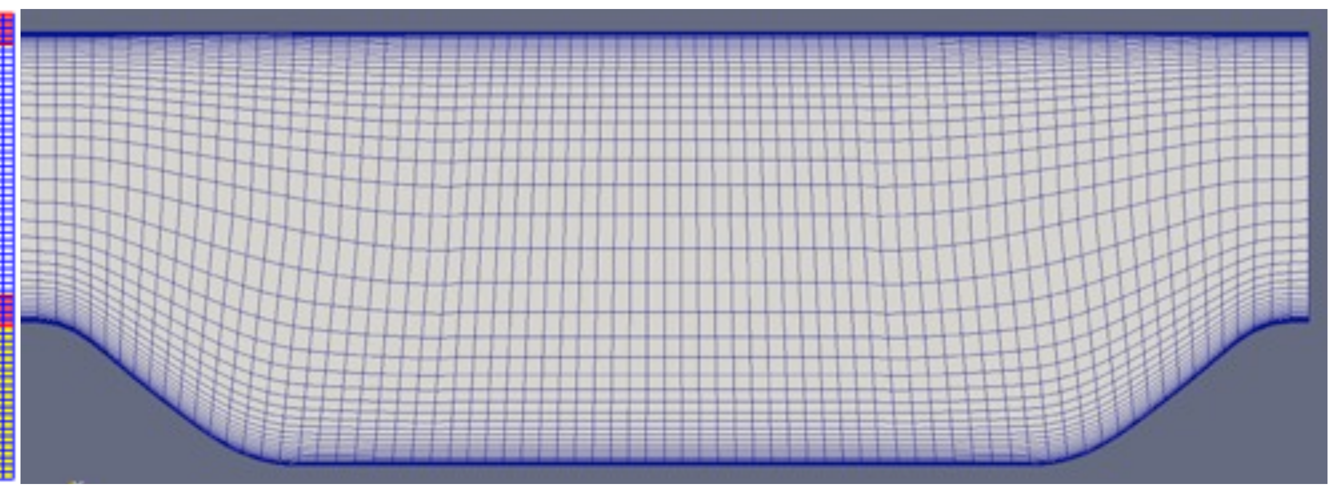


RANS on body-fitting grid



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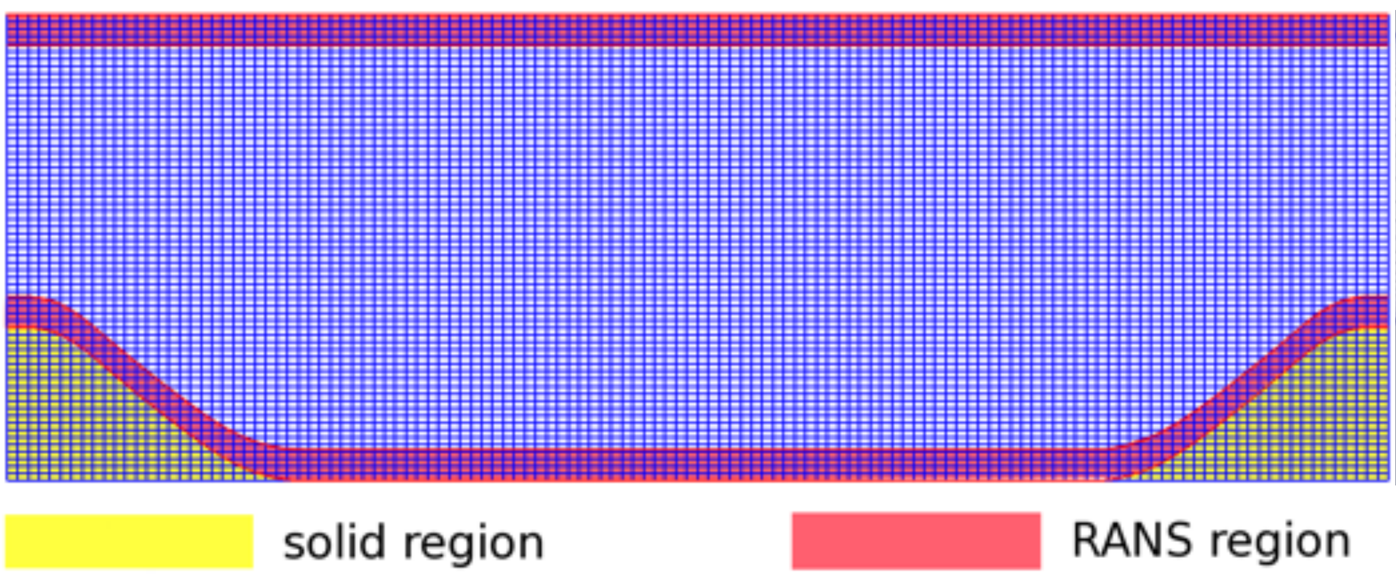
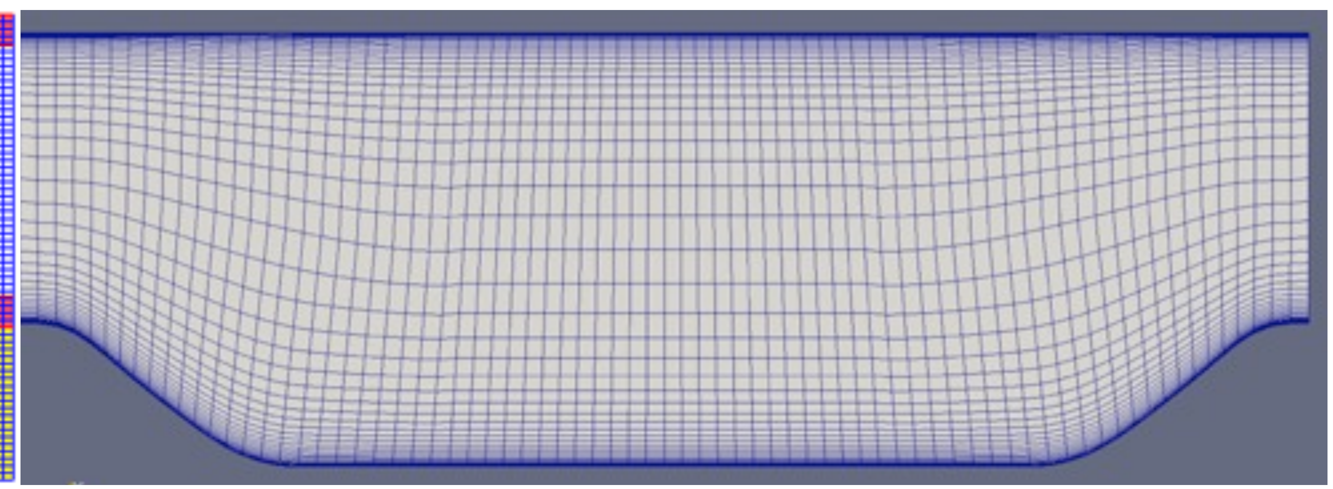


RANS on body-fitting grid



Note: 1. Accuracy of immersed boundary forcing is not critical.
2. On Cartesian grid, stretching towards curved boundaries is not easy => no stretching in our LES!

IMPACT/OpenFOAM Coupling: Numerics

LES solver: IMPACT

- 6th-order compact finite difference scheme
- Cartesian mesh (with possibility of stretching)
  Staggered grid + Explicit time-stepping (RK3)
- ADM-RT for Sub-Grid Scale modeling

Approximate **D**econvolution **M**odel with **R**elaxation **T**erm

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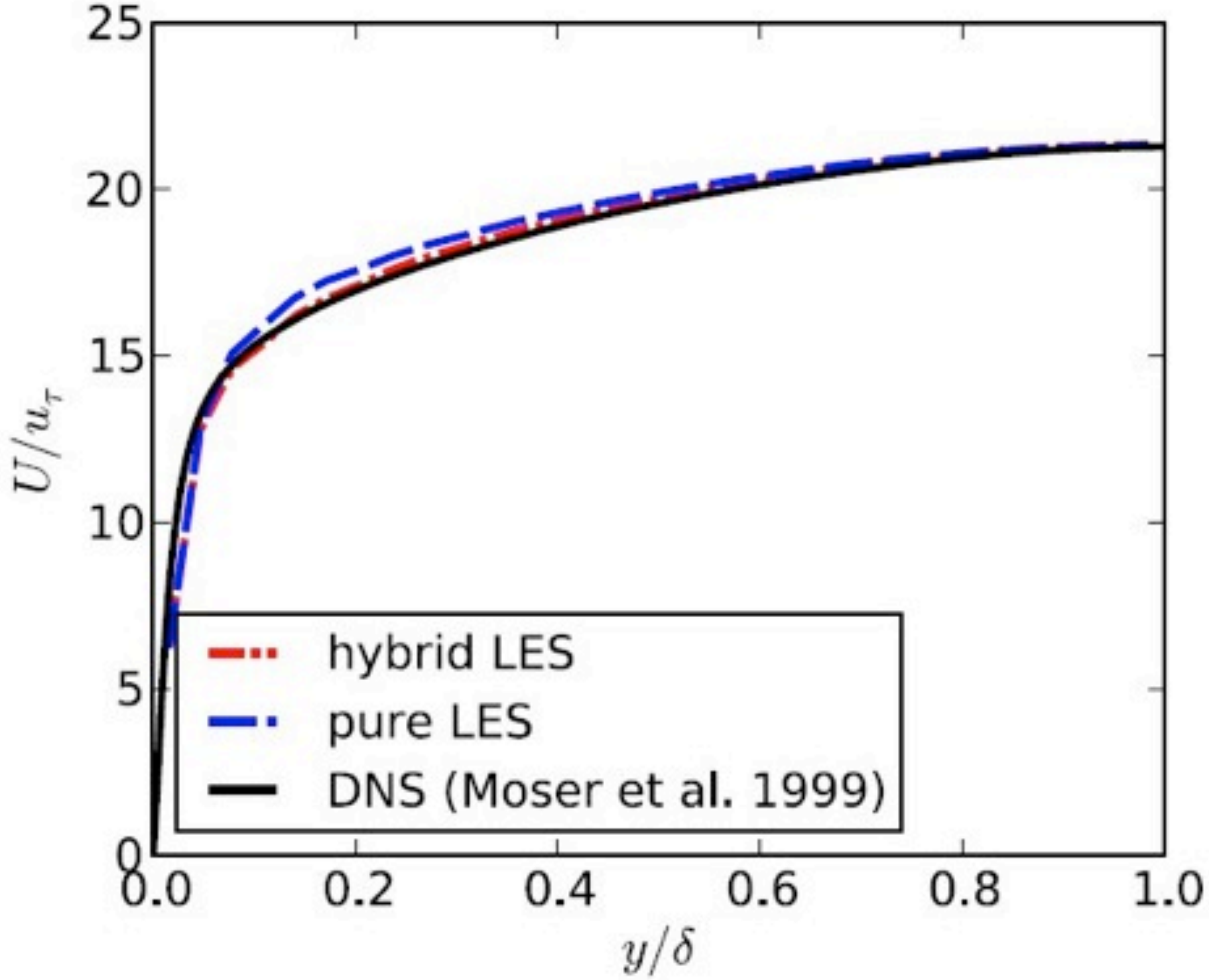
RANS solver: OpenFOAM (open-source CFD platform)

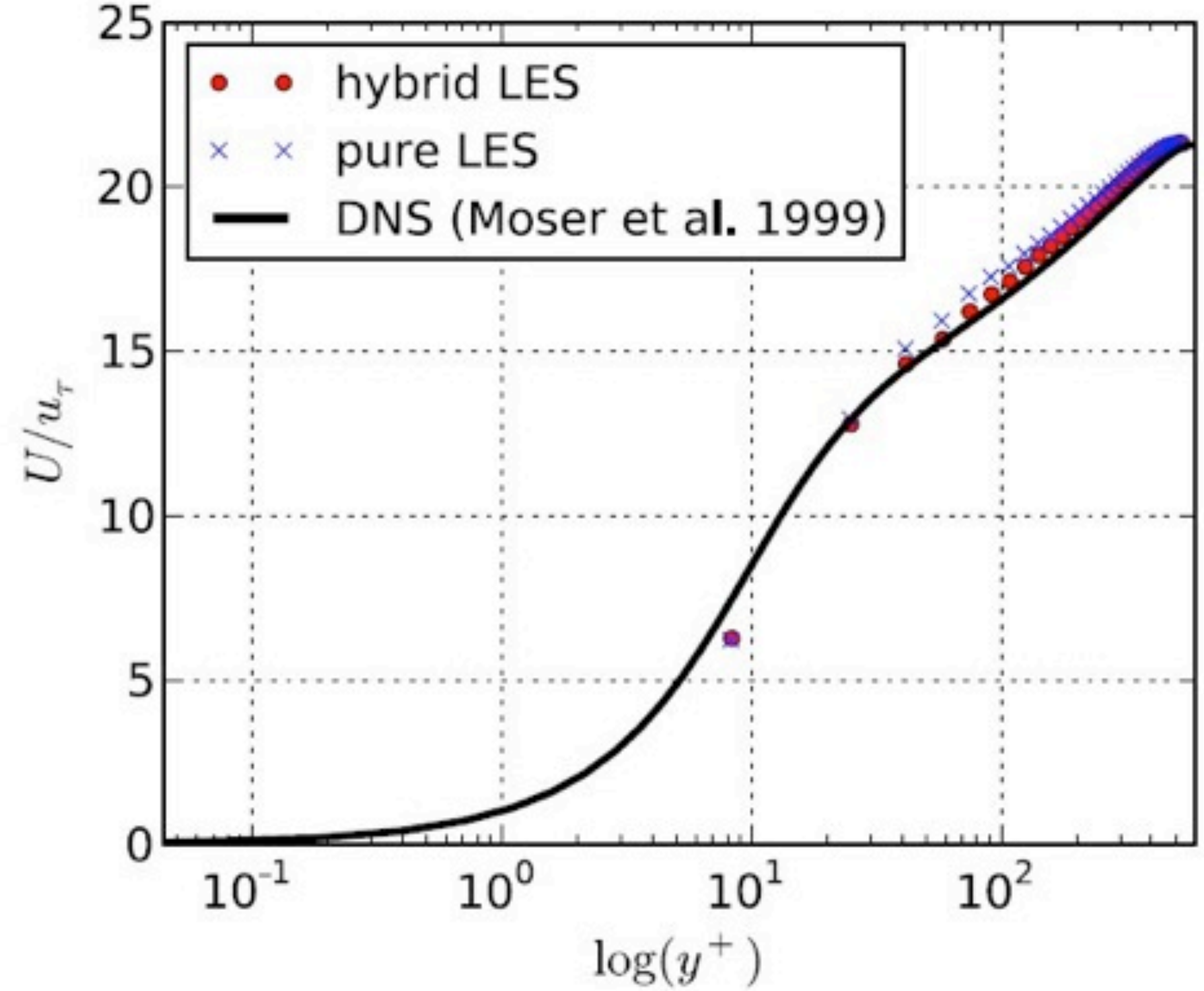
- Second-order finite volume discretization
- Body-fitting mesh (globally unstructured); Co-located grid + PISO algorithm
- A variety of common RANS turbulence models

Periodic Channel Flow $Re_\tau = 590$

Outer Coordinate



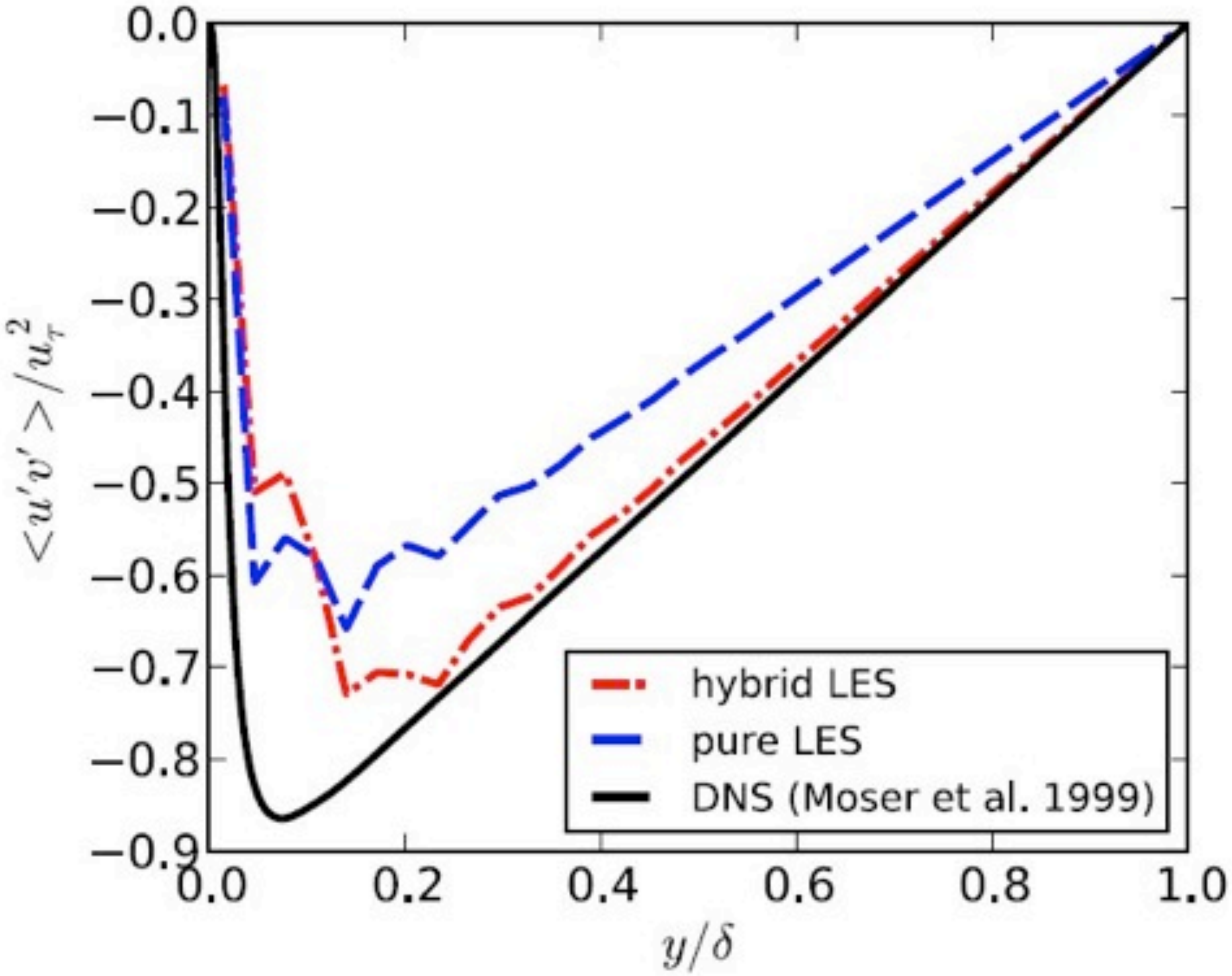
Inner Coordinate



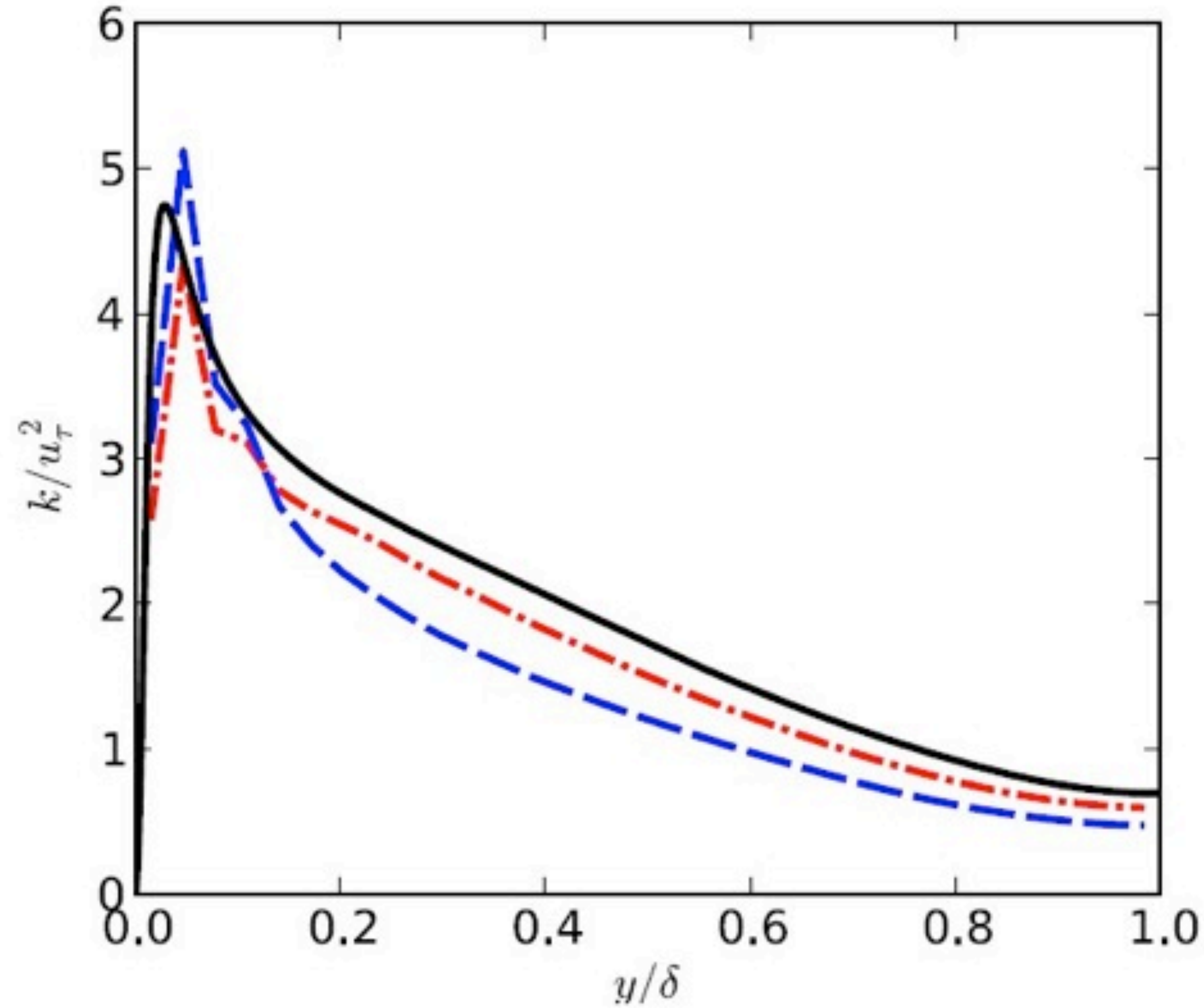
- Improved prediction; No log-layer mismatch

Periodic Channel Flow $Re_\tau = 590$

Turbulent shear stress



TKE

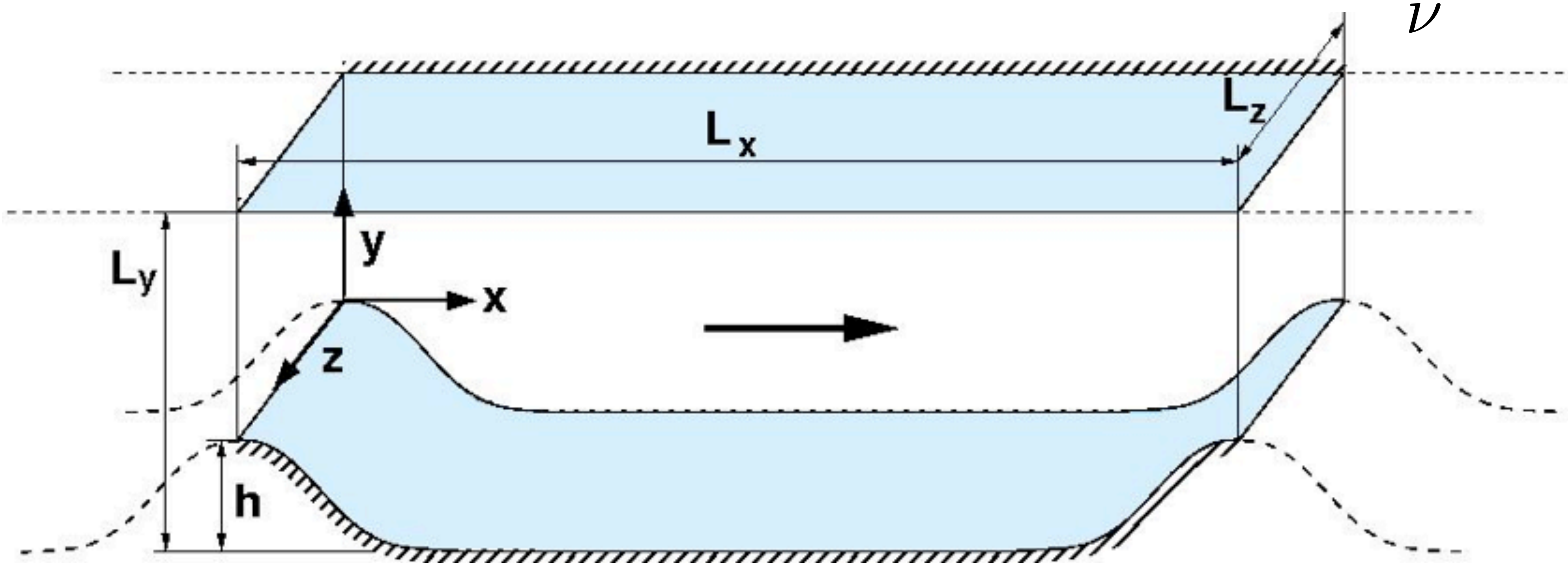


- Significantly better shear stress & TKE in the outer layer
- SGS contribution not know (due to the nature of the ADM-RT model)

Periodic Hill: Problem Setup

- Periodic BCs in streamwise + spanwise directions.
- Pressure gradient to maintain constant mass flux.
- Reynolds number based on hill height and bulk velocity at crest: 2800 and 10595

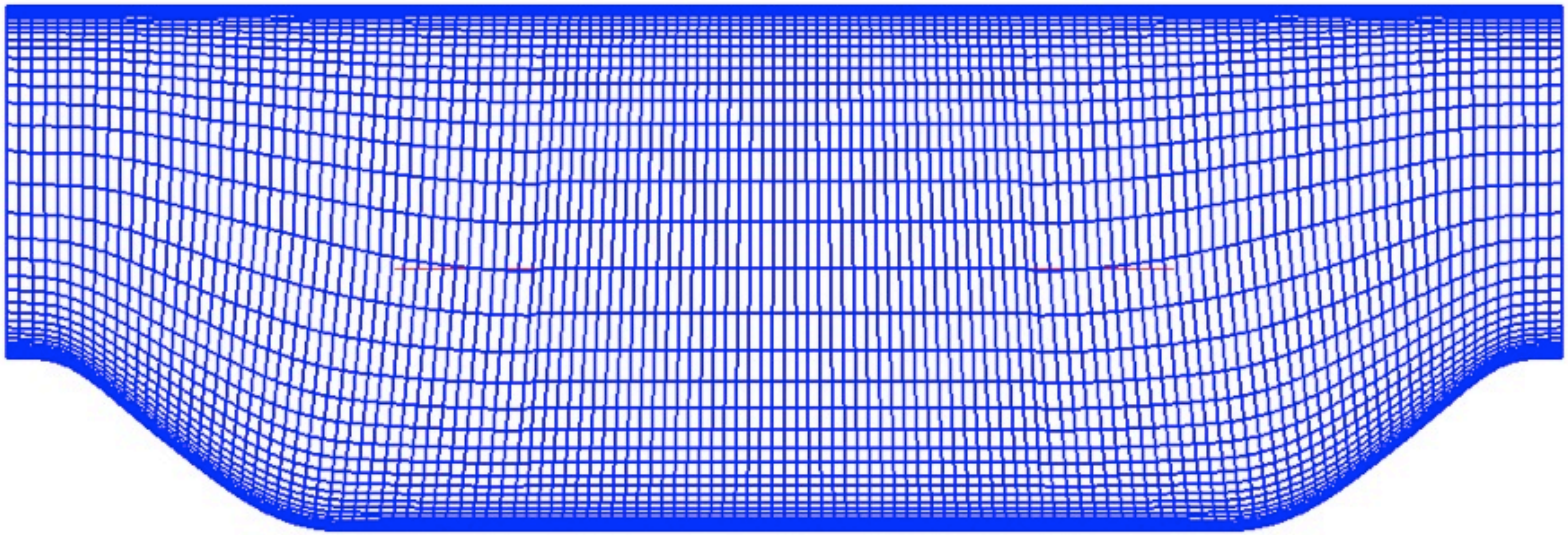
$$Re = \frac{HU_b}{\nu}$$



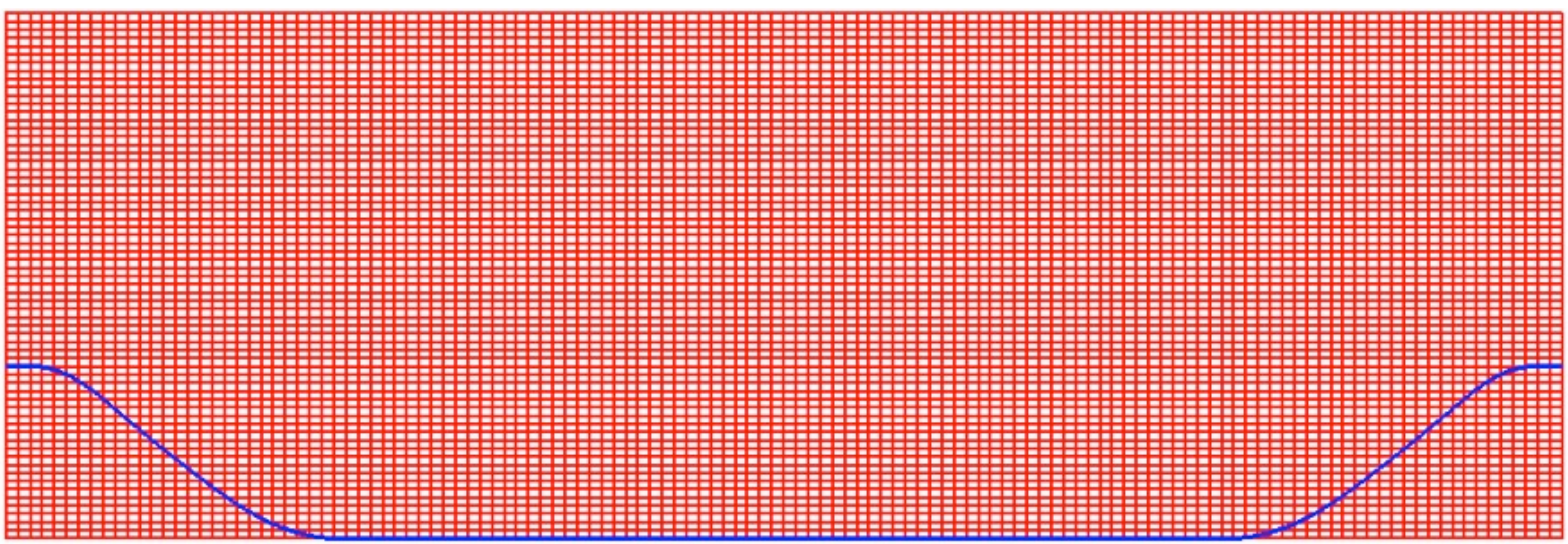
Rapp, http://qnet.cfms.org.uk/index.php?title=Image:Hill3d.jpg

Different Domain / Inconsistent Meshes

RANS



LES



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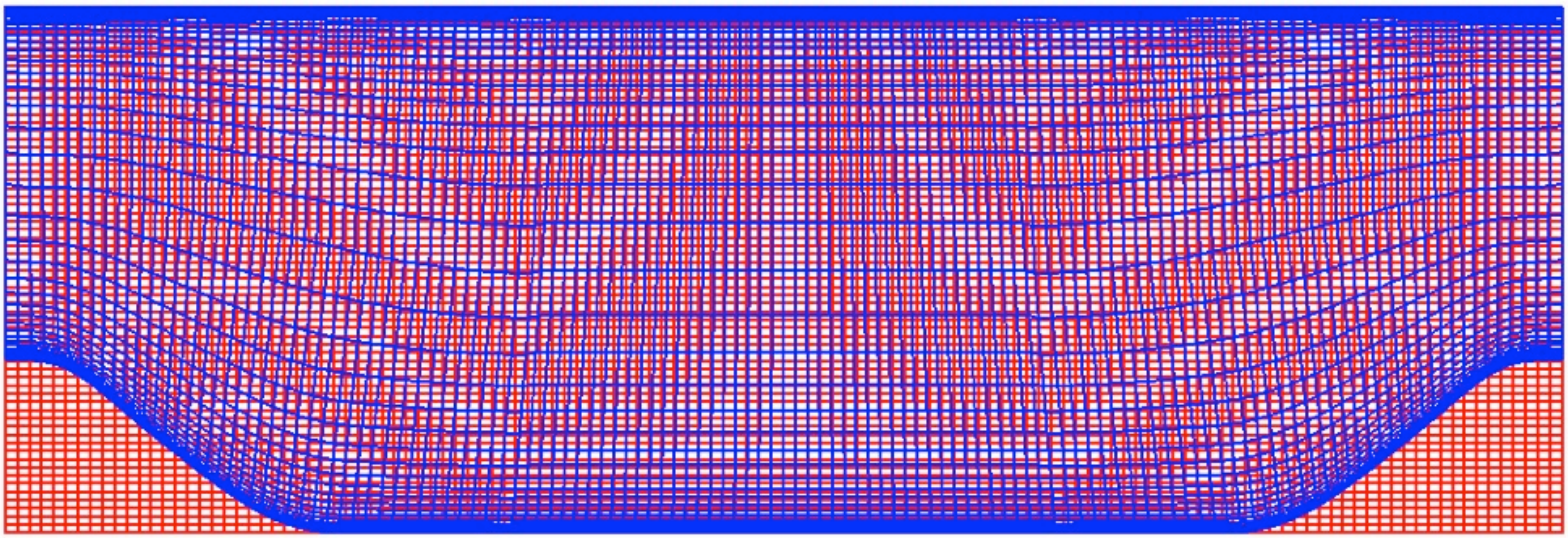
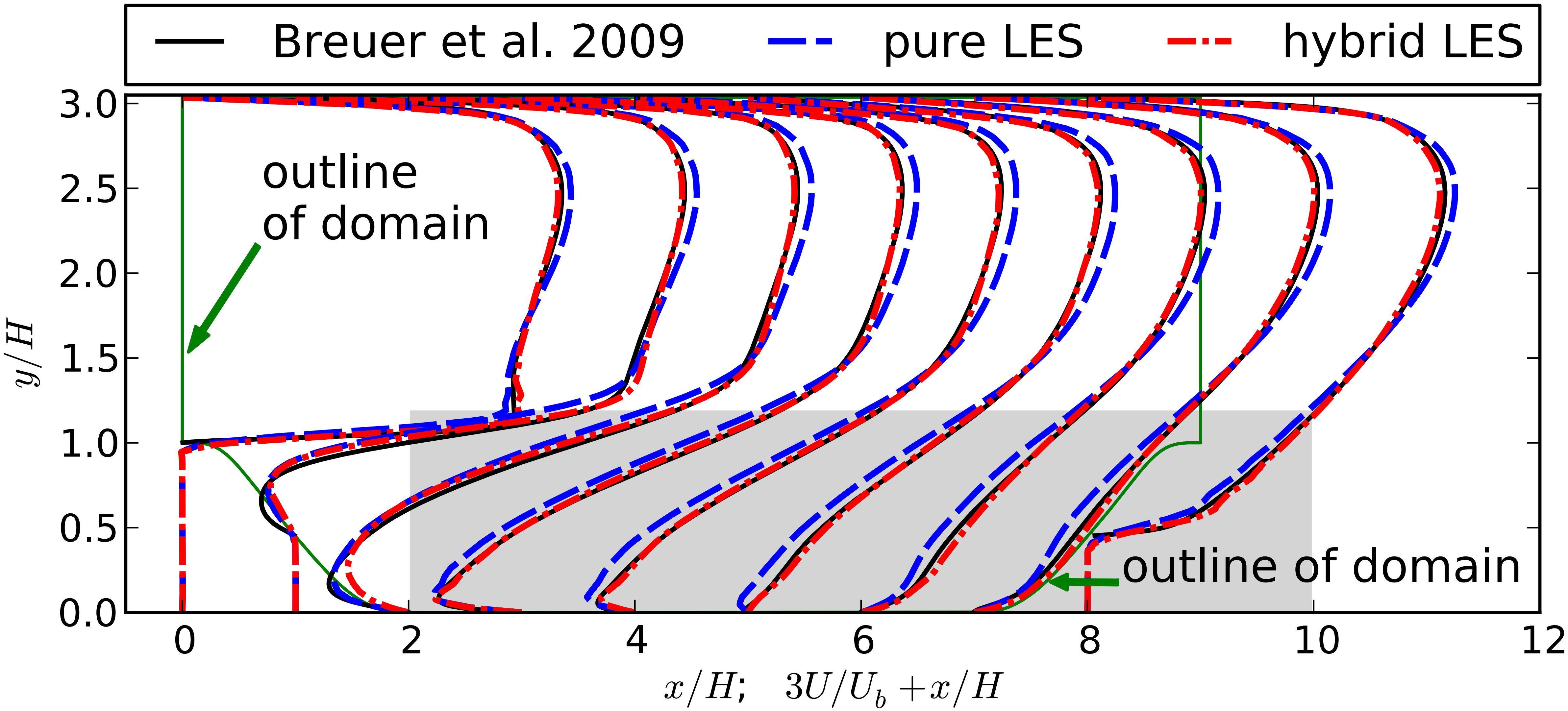


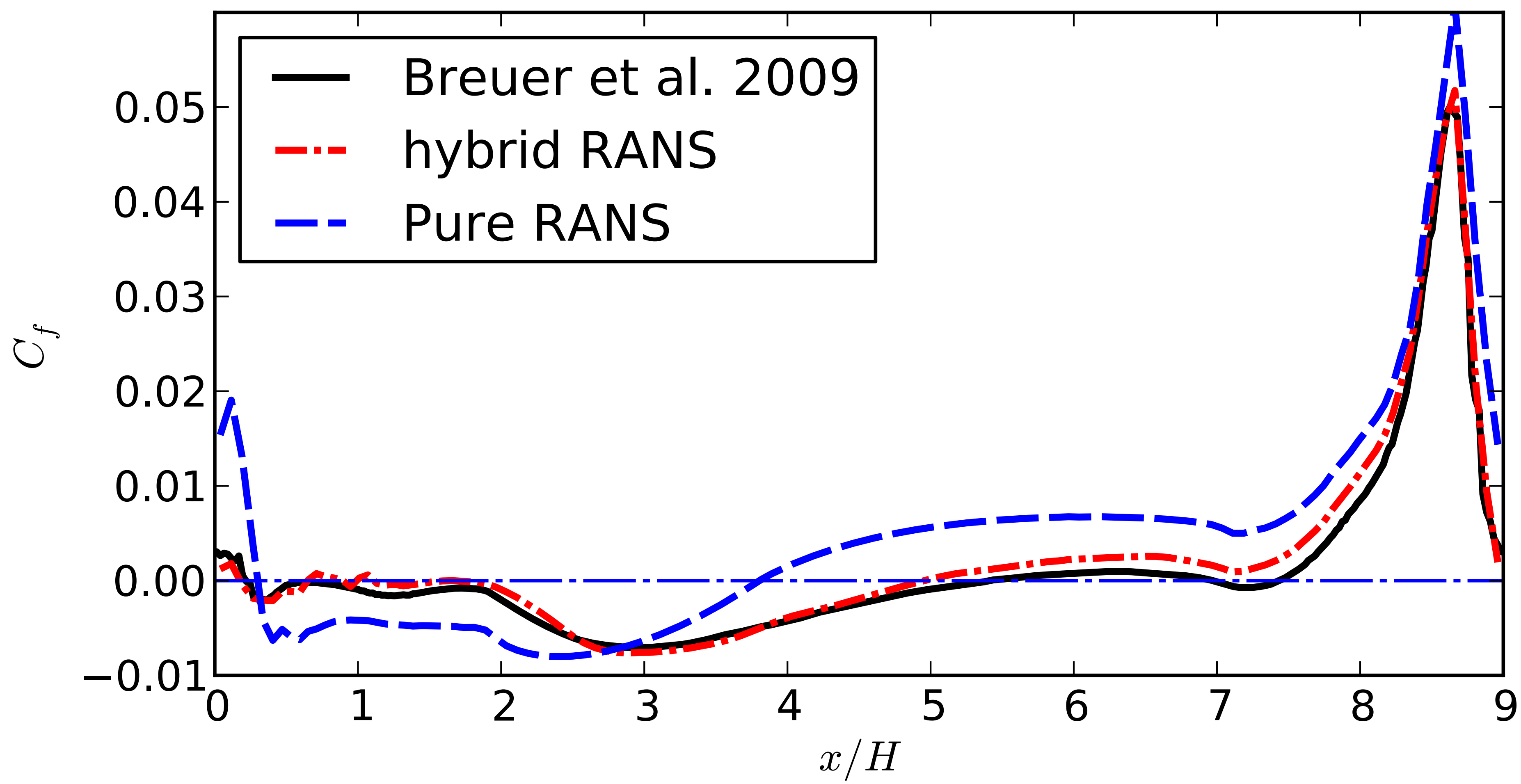
Velocity Profiles for Re=2800



$y^+=9$

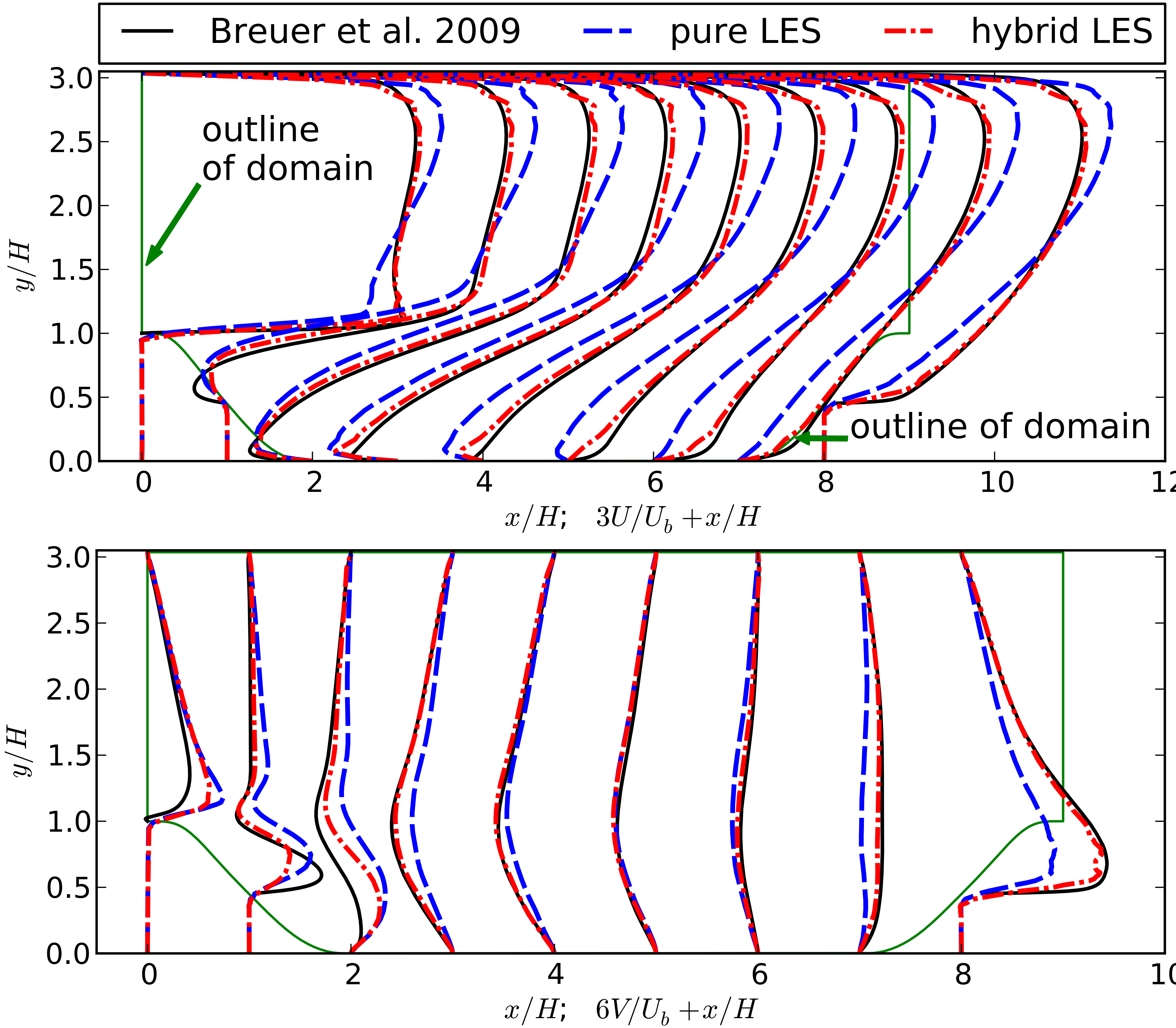
- Pure LES is quite good: boundary forcing OK!
- Improvement of hybrid solve in free shear region and in the reattached region (shaded).

Wall Shear Stress for Re2800



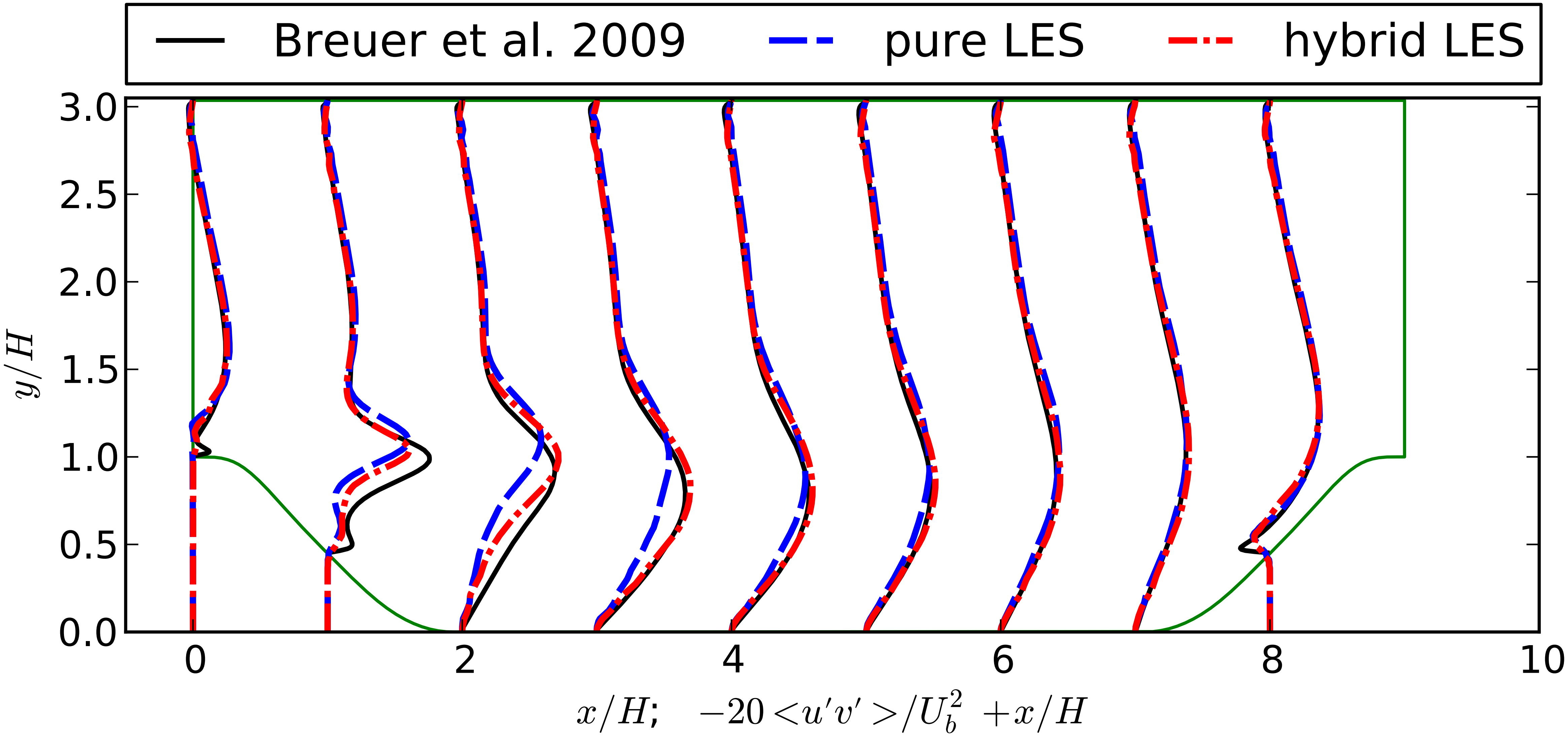
- Good prediction of wall shear stress: LES captured the overall behavior + RANS captured near-wall velocity profile
- Good prediction of separation/reattachment by hybrid LES/RANS, but not by pure RANS!

Velocity Profiles for Re=10595

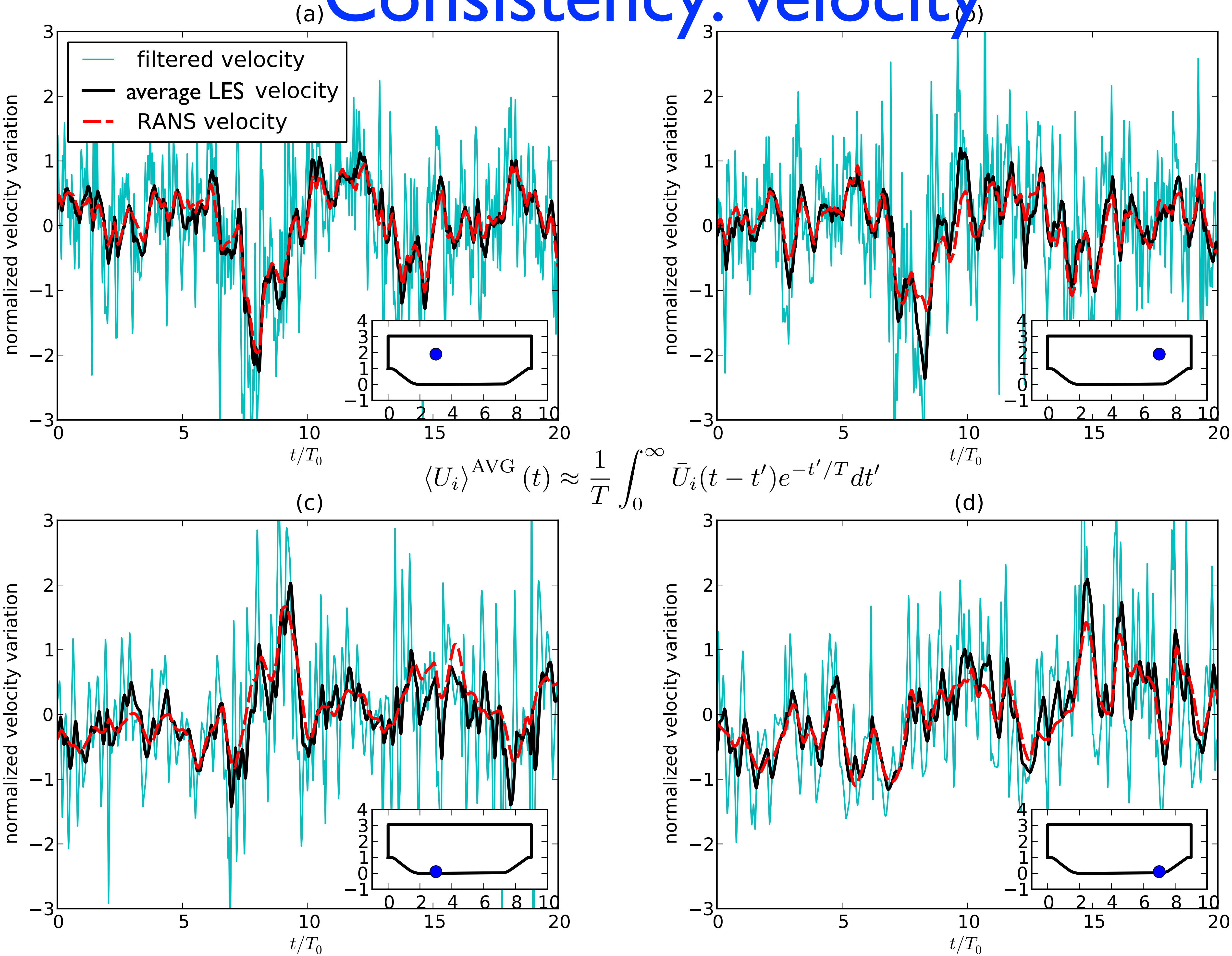


y^+=27

Comparison of Turbulent Shear Stress

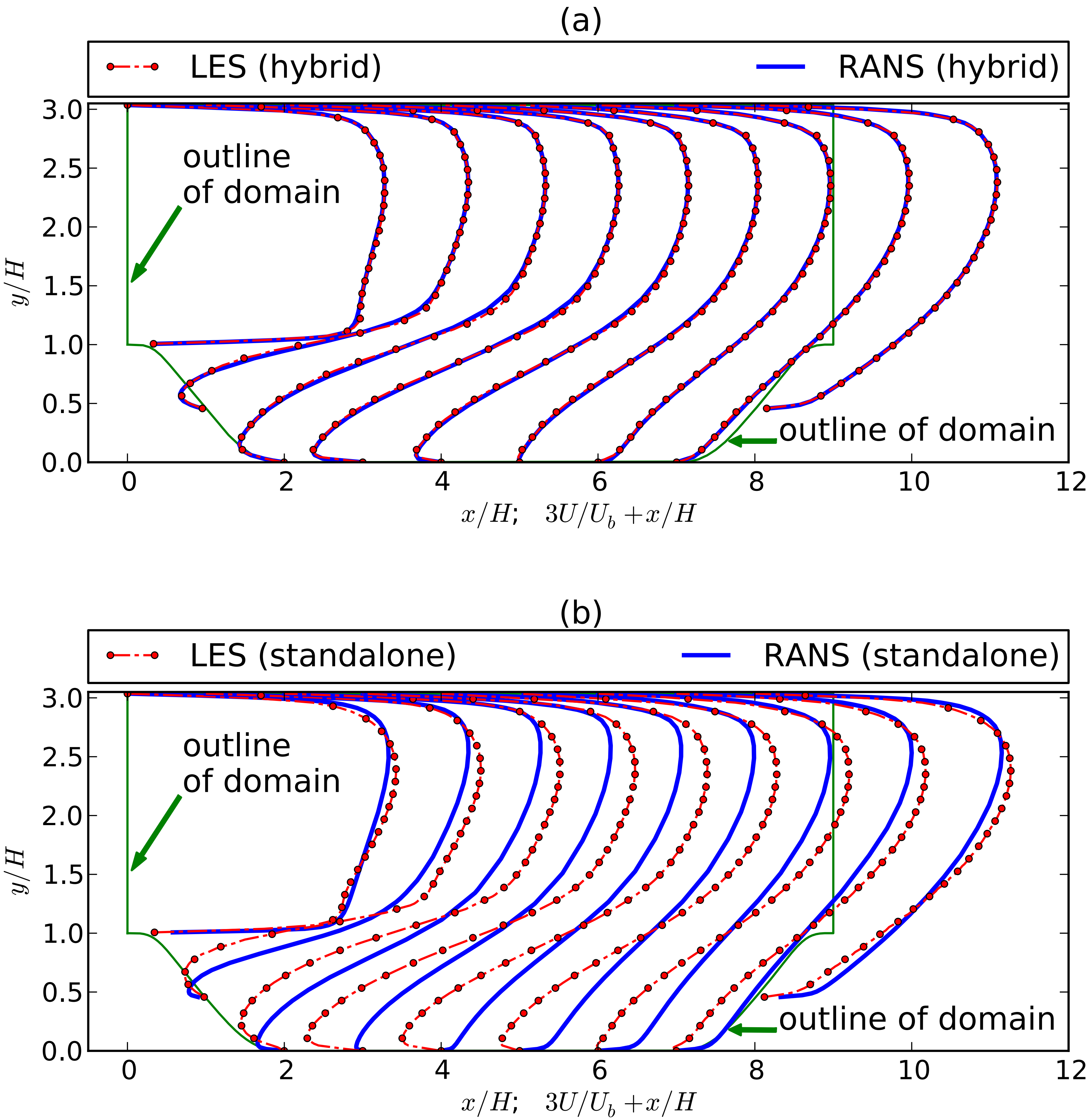


Consistency: velocity

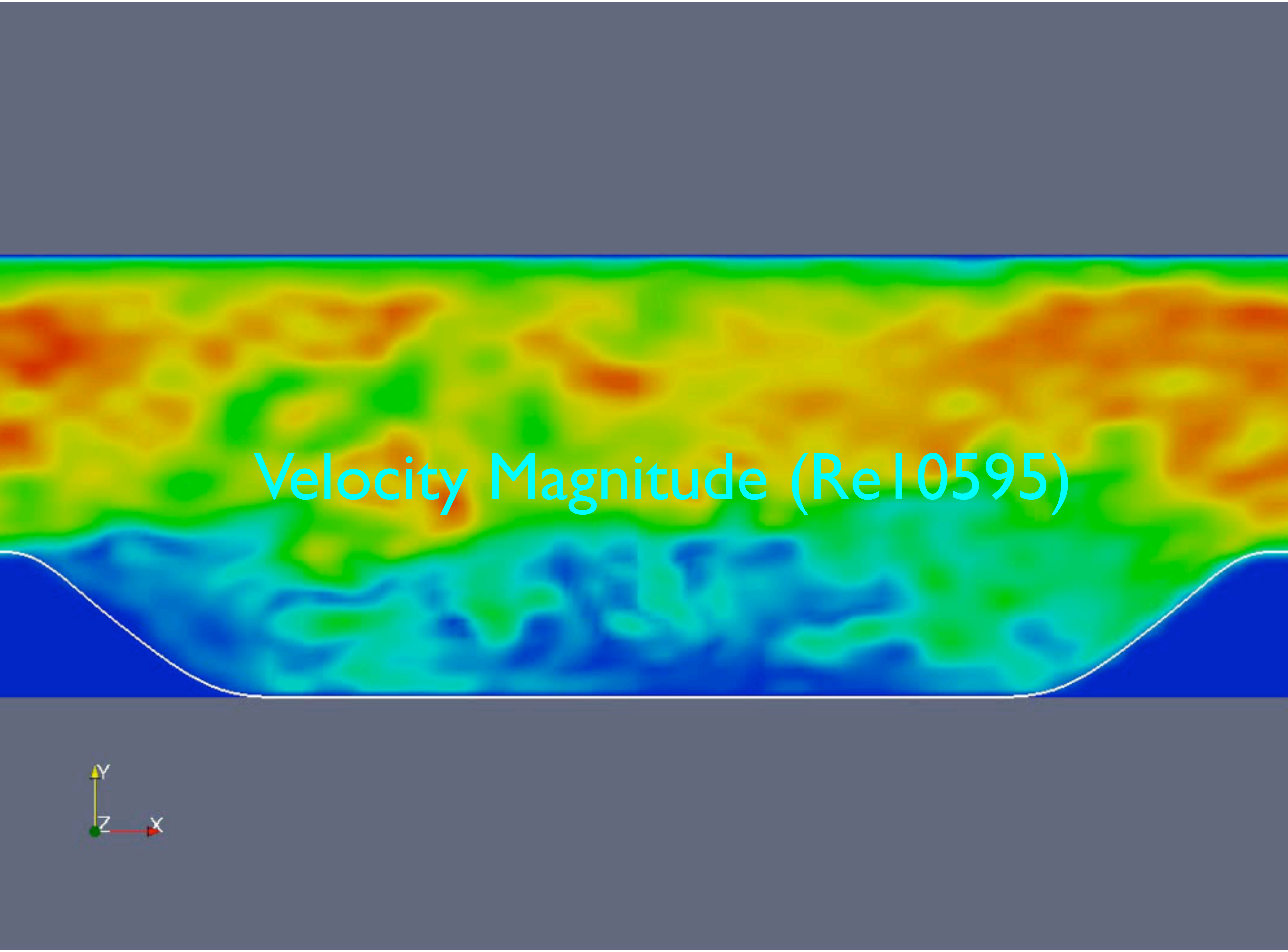


$$\langle U_i \rangle^{\mathrm{AVG}}(t) \approx \frac{1}{T}\int_0^\infty \bar{U}_i(t-t')e^{-t'/T}dt'$$

Consistency: Mean Velocity



No coupling


Velocity Magnitude (Re10595)
Y
Z
X

Conclusion and Outlook

- We introduced a dual-mesh consistent hybrid LES/RANS framework where LES and RANS are conducted simultaneously for the same flow.
- A coupled solver using high-order LES on Cartesian grid and RANS on body-fitting grid has been developed, using an immersed boundary technique.
- The hybrid solver performs better than under-resolved pure LES (on the same mesh).
- The framework can explore the potential of existing academic LES codes for complex flows.

Thank you for you attention!

Questions?