Solution-adjusted Formulation of Vorticity Confinement with Applications to Tip Vortices of Stationary and Rotating Wings

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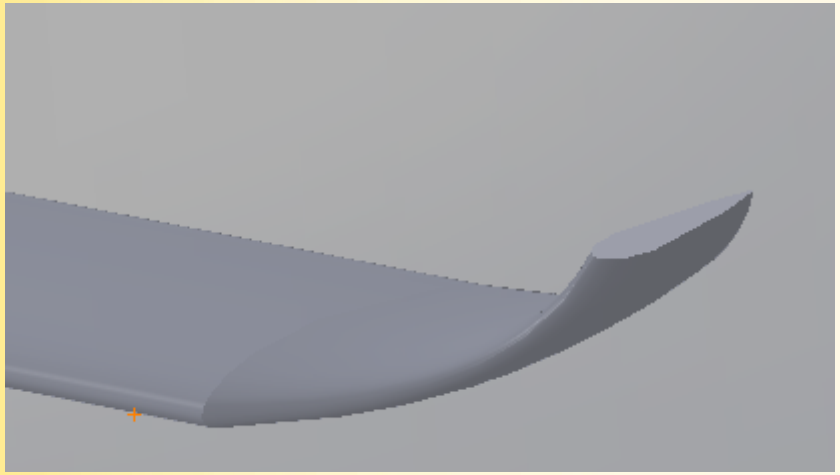
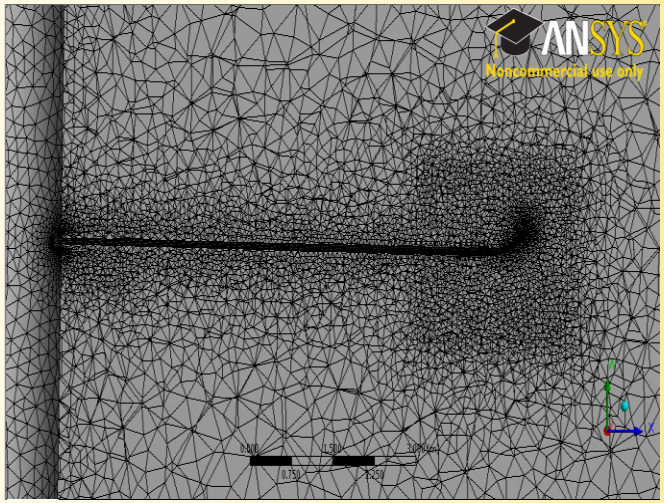
Overview

Motivation: Increase accuracy/reduce grid size for CFD modeling of airflow around blades and wings

Example: helicopter blade winglet design



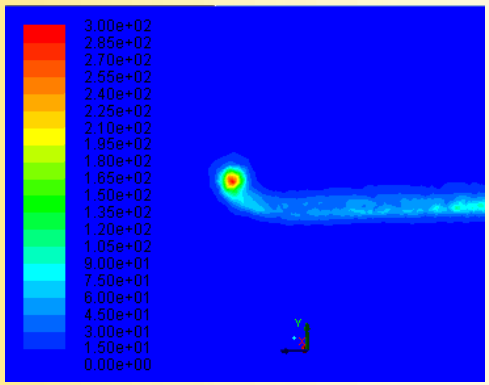
overall view of winglet geometry



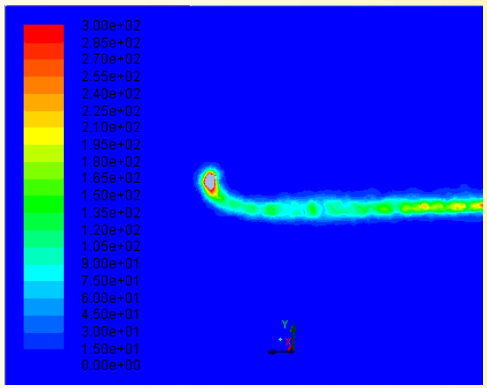
generated mesh around rotor-blade-winglet configuration

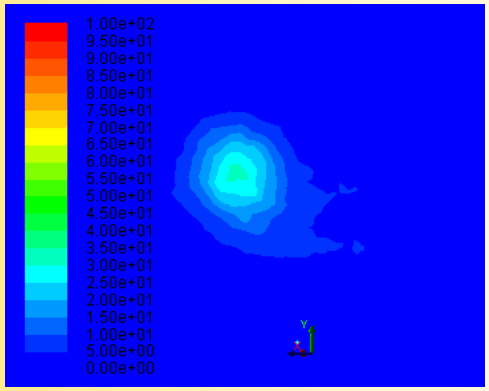
Role of tip vortex: lift, drag, interaction of tip vortex with next blade, sound, vibration

Role of discretization in tip vortex decay
1st order upwind vs 2nd order upwind

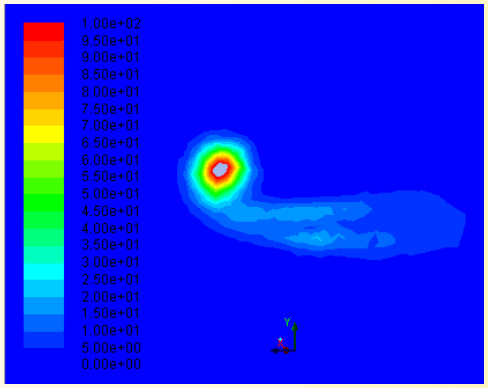


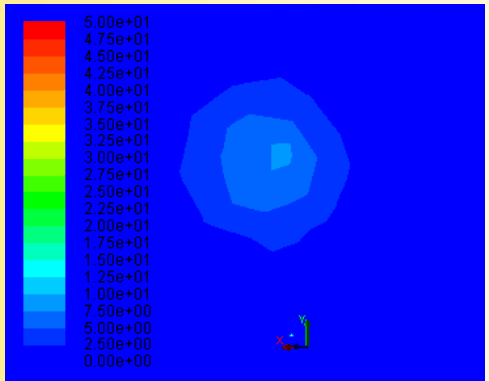
5 degrees



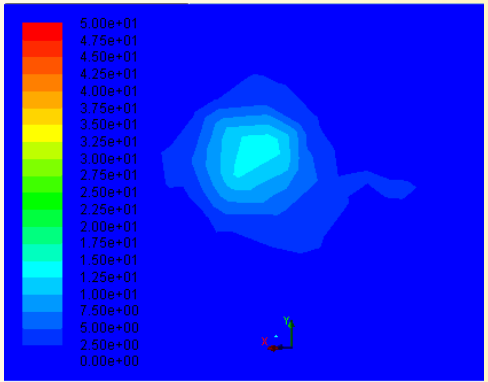


30 degrees





60 degrees



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Upwind Discretization Issues

- Many CFD solvers employ an upwind discretization for convective terms
- While improving stability, this method comes at the cost of increased numerical dissipation
- As they convect downstream, vortices are smeared and, for example, the body-vortex interaction or induced drag are not predicted adequately.
- Numerical anti-diffusion technique called **Vorticity Confinement** is introduced by John Steinhoff, University of Tennessee Space Institute, in the late 1980s

Vorticity Confinement (VC) Method

- VC captures vortices concentrated over few grid cells as they convect downstream through the addition of body force terms

- VC used as a means to improve the results obtained by finite-volume upwind CFD solvers without the computational cost of very refined grid

- *Q: How does VC work?*
  A: VC adds the highlighted source term to the momentum equation (Steinhoff, 1994; Lohner, 2002)

$$\frac{\partial \vec{q}}{\partial t} = -(\vec{q}\cdot\nabla)\vec{q} - \frac{1}{\rho}\nabla P + \mu\nabla^2\vec{q} - \boxed{\varepsilon\vec{s}} \qquad \vec{n} = \frac{\nabla|\vec{\omega}|}{\left|\nabla|\vec{\omega}|\right|}$$

$$\vec{s} = \vec{n}\times\vec{\omega} \qquad \varepsilon = ch^2\,|\nabla|\varpi||$$

Background: Vorticity Confinement

$$\frac{\partial \vec{q}}{\partial t} = -(\vec{q}\cdot\nabla)\vec{q} - \frac{1}{\rho}\nabla P + \mu\nabla^2\vec{q} - \varepsilon\vec{s} \qquad \vec{n} = \frac{\nabla|\vec{\omega}|}{\left|\nabla|\vec{\omega}|\right|} \qquad \vec{s} = \vec{n}\times\vec{\omega}$$

The VC has following variants:

-original (constant confinement parameter ε)

-modified (constant unit-less confinement parameter c) $\varepsilon = ch^2\,|\nabla|\varpi||$

VC needs heuristic problem-dependent parameter (either c or ε)

Adjusted vorticity confinement (AVC) formulation by Hahn and Iaccarino (2009)

Evaluates the needed magnitude of VC by the difference between central and upwind discretizations. The algorithm is *in general* numerically unstable and needs threshold.

Finite-volume (FV) approach to momentum conservation equation:

$$\frac{q_i^{n+1} - q_i^n}{\Delta t} = \frac{F_{i+\frac{1}{2}} - F_{i-\frac{1}{2}}}{\Delta x} - \epsilon \vec{s}$$

For the Steger-Warming first-order flux vector splitting scheme, the flux at the FV interface become:

$$F_{i+\frac{1}{2}} = F_i^+ + F_{i+1}^-$$

The flux at the interface can be extended to second-order accuracy using a Total Variation Diminishing (TVD) limiter:

$$F_{i+\frac{1}{2}} = F_i^+ + 0.5\Phi(F_i^+ - F_{i-1}^+) + F_{i+1}^- + 0.5\Phi(F_{i+1}^- - F_{i+2}^-)$$

$$\Phi_{i+1/2}^+ = f((F_{i+1}^+ - F_i^+)/(F_i^+ - F_{i-1}^+)$$

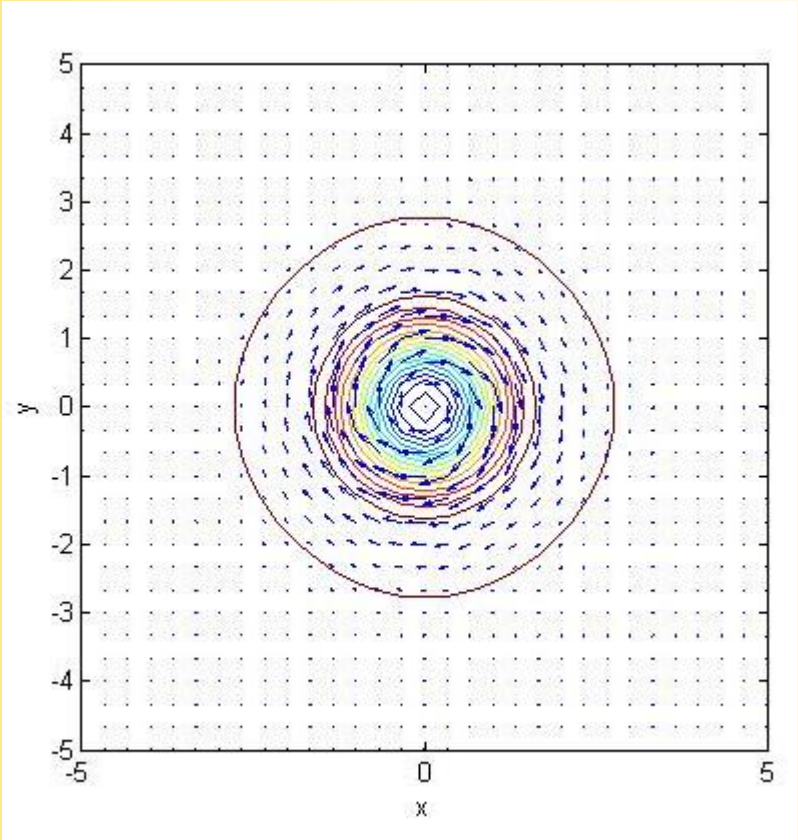
Coupling of TVD and VC

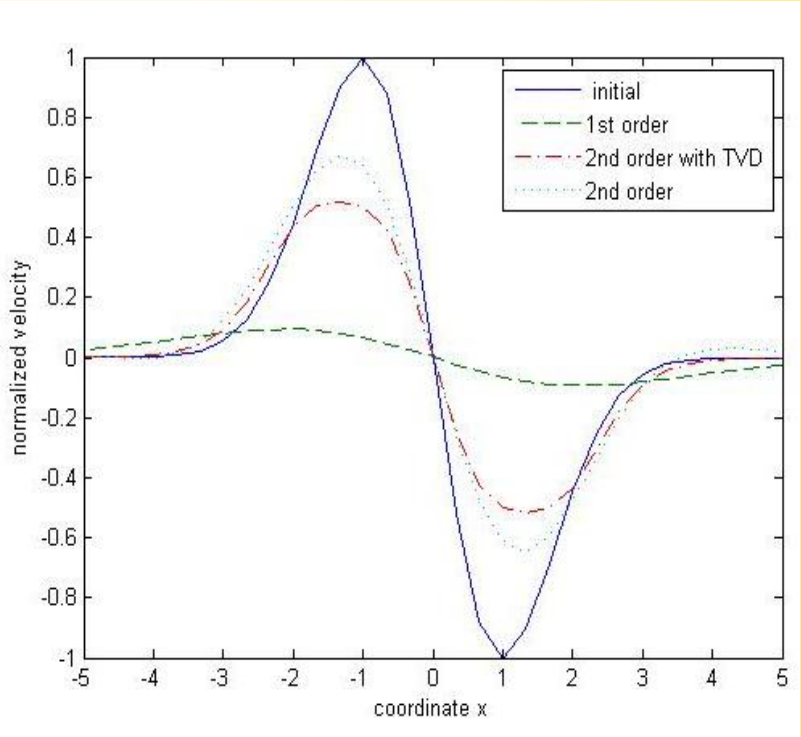
- Coupling of Total Variation Diminishing (TVD) with either VC or AVC could eliminate the heuristic parameter (ε or c)
(Povitsky and Pierson, 2013; Vatistho and Povitsky, SBIR Report, 2014)

Objectives:

- To have self-adjusting VC/TVD algorithm to avoid over-confinement and use of heuristic parameter.
- Examine combination of VC with the second-order upwind TVD scheme for various flux limiters and select the best limiter.
- Validate the approach for convected Taylor vortex.
- Apply the proposed VC/TVD scheme to CFD model of tip vortex of helicopter's rotating blade and stationary wing and compare to experiments.

2-D convection of Taylor vortex





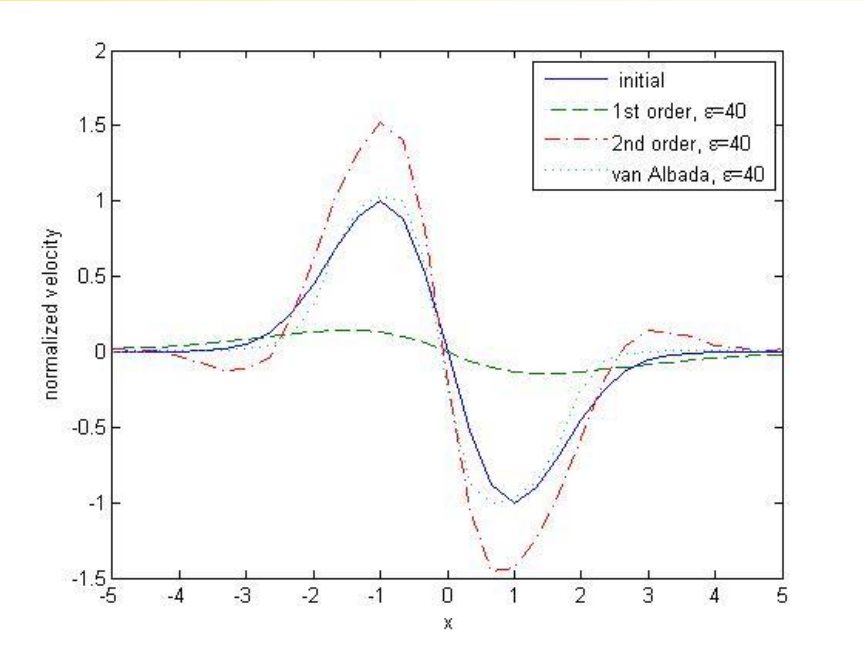
The domain is a rectangle *[-10<x<40] x [-25<y<25]*.
The uniform grid of 150 x 150 is taken that corresponds to *Δx=Δy=1/3*.
There are only 6 grid intervals per vortex diameter.
The vortex is convected 200 time steps with the CFL value equal to 0.4.

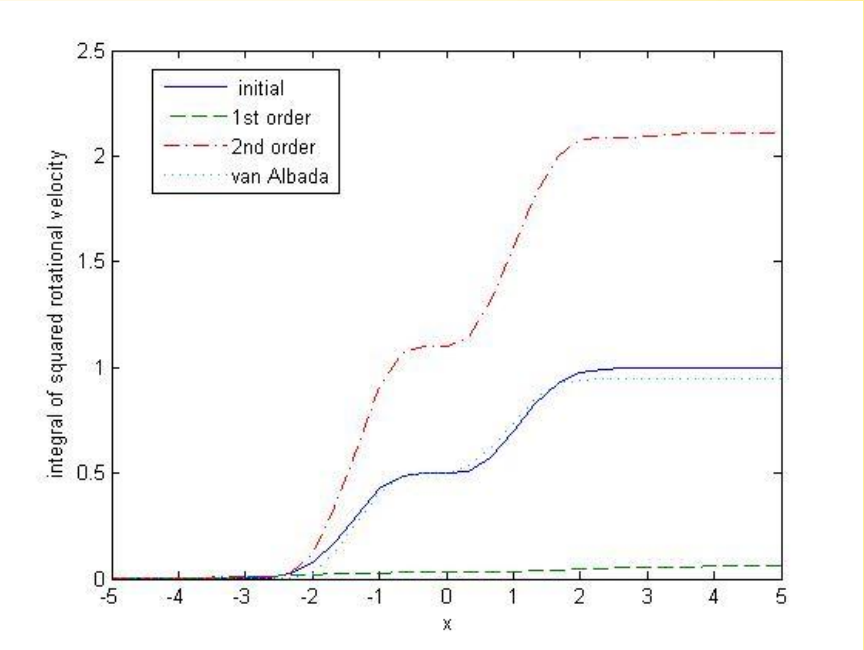
VC formulation with constant parameter *ε*
Convection of Taylor vortex

first-order upwind scheme,
second-order upwind scheme
and second-order scheme with TVD.

The integral of squared rotational velocity

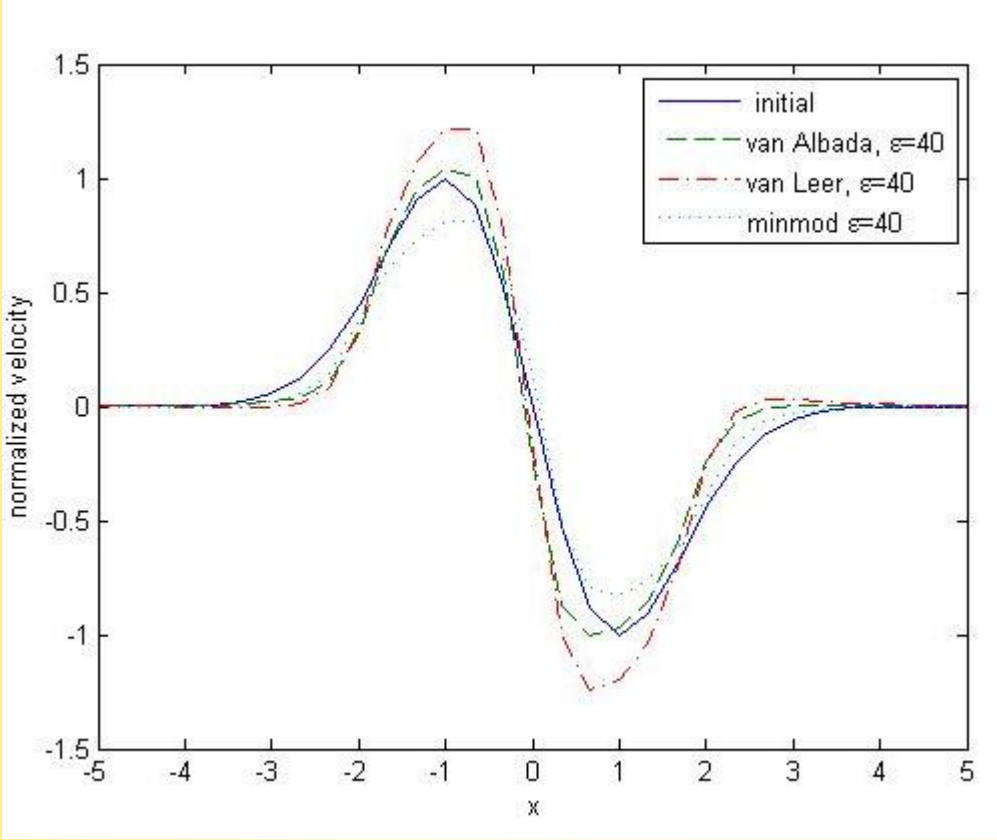
$$I = \int r u^2 dr$$





The physical meaning of ***I*** is dynamic pressure of vortex that is important for vortex impingement into next blade.

Combination of TVD with Vorticity Confinement: comparison of TVD limiters



Minmod:

$$f(x) = \max(0, \min(1, x))$$

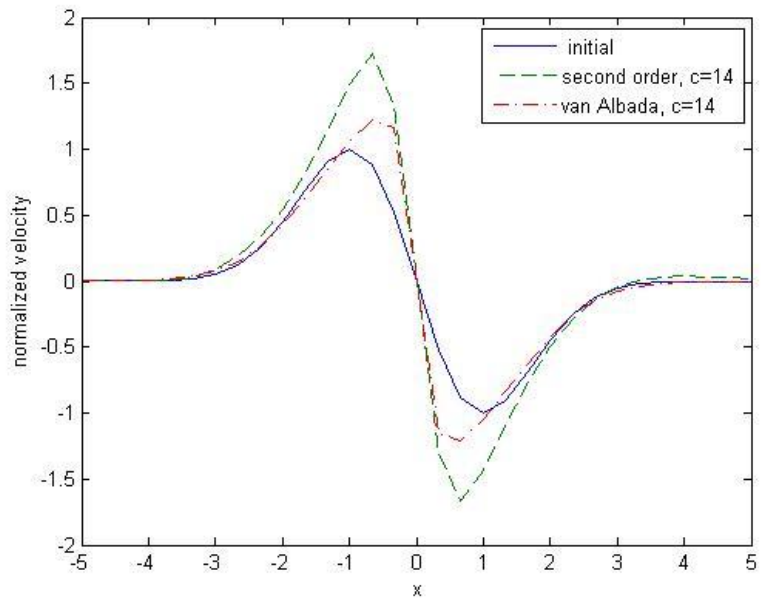
Van Leer:

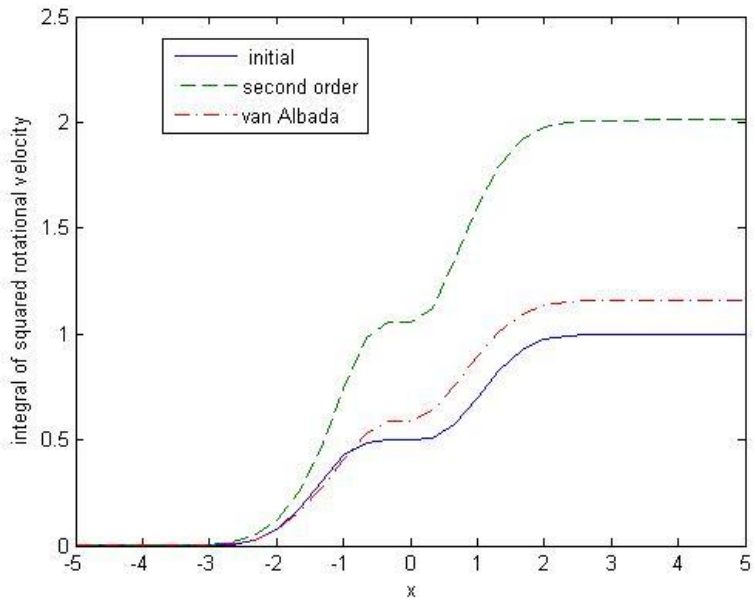
$$f(x) = (x + |x|)/(1 + |x|)$$

Van Albada:

$$f(x) = (x^2 + x)/(x^2 + 1)$$

Modified Formulation of Vorticity Confinement, constant unit-less parameter *c=14*





second-order upwind scheme and second-order scheme with TVD (van Albada limiter).

The integral of squared rotational velocity

2D Simulation of Vortex with VC TVD Scheme Evaluation

- Steger-Warming flux splitting
- 2nd -order upwind approximation of fluxes created through flux extrapolation
- The initial vortex velocity profile is given by:

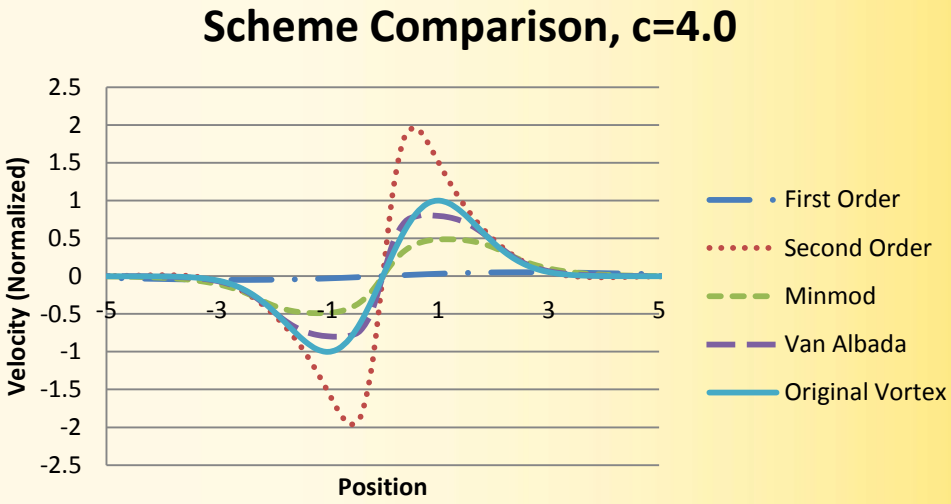
$$U = r\frac{\Gamma}{2\pi}\exp\left(0.5(1-r^2)\right)$$

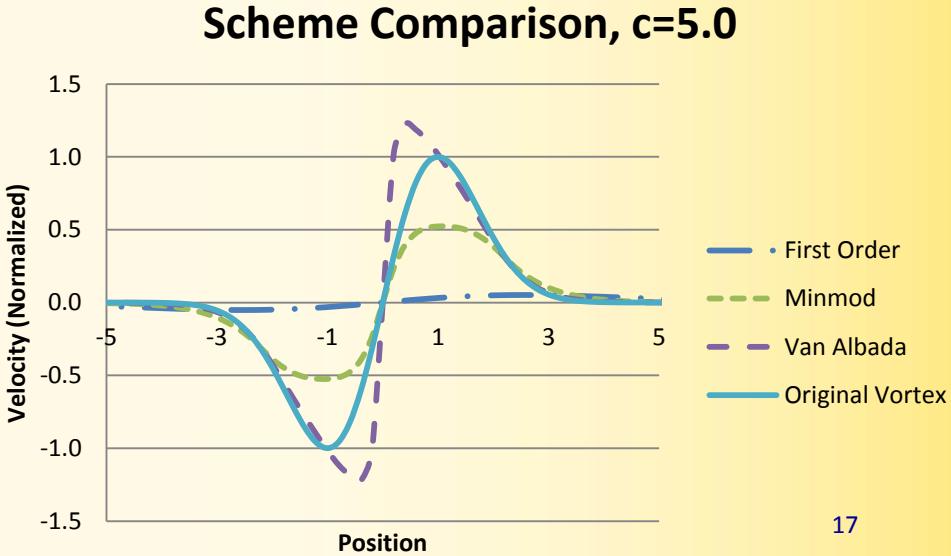
$$u = -U * \frac{y}{r}\ ,\ v = U * \frac{x}{r}$$

- Inviscid
- Grid:
  - $\Delta x = \Delta y = 0.2$
  - Vortex Core radius ≈ 1 meter, 10 steps across vortex diameter
- Simulation run for total time, t=0.1s corresponding to 2000 iterations

TVD Results

- The minmod scheme results in the highest level of numerical stability and lower dependence on confinement parameter
- However, the minmod scheme suffers from excessive numerical dissipation
- The Van Albada limiter improved stability over the purely second-order scheme; it also experienced less numerical dissipation compared to the Minmod limiter.





Adjusted formulation by S. Hahn and G. Iaccarino

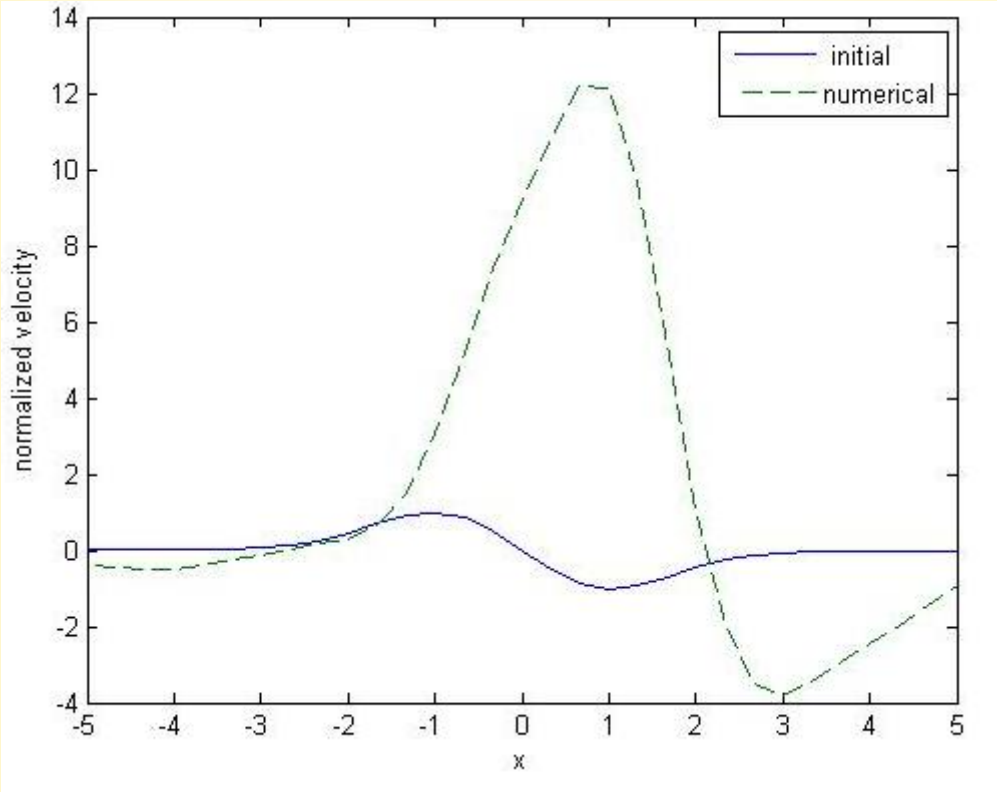
For the adjusted VC formulation (AVC), neither values of c and ε are needed. Instead the value of parameter ε is determined by the difference of the central and upwind approximations of fluxes. For 2-D vortex convection:

$$\varepsilon = \frac{D_1 n_2 - D_2 n_1}{\omega}$$

where *D1* and *D2* are differences between central and upwind fluxes in the *x* and *y* directions

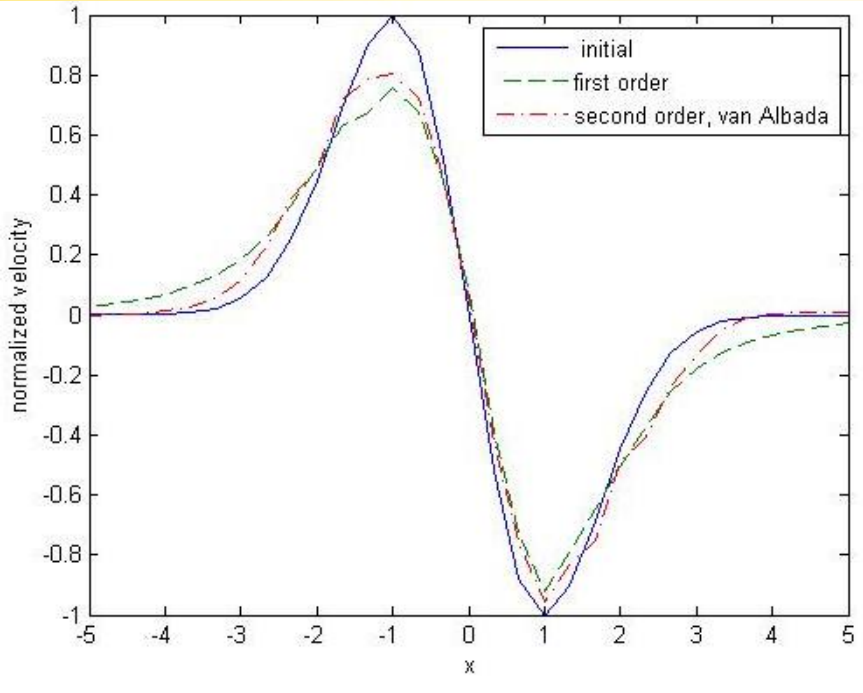
S. Hahn and G. Iaccarino,"Towards Adaptive Vorticity Confinement," *47th AIAA Aerospace Sciences Meeting and Aerospace Exposition*, AIAA 2009-1613, Orlando, Florida, 2009

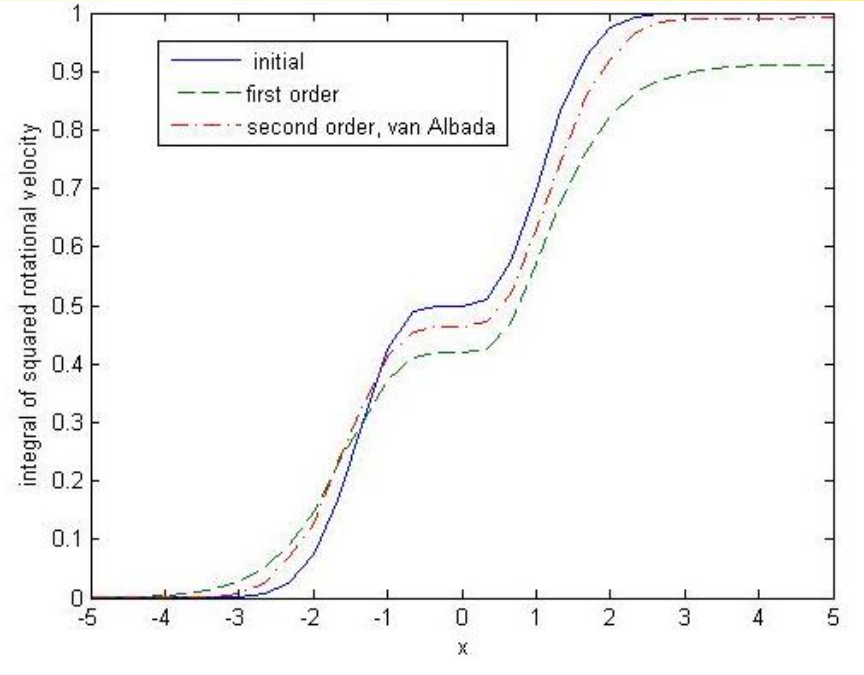
Second-order upwind scheme with the AVC formulation. The vortex is dramatically over-confined:



where

AVC combined with TVD:

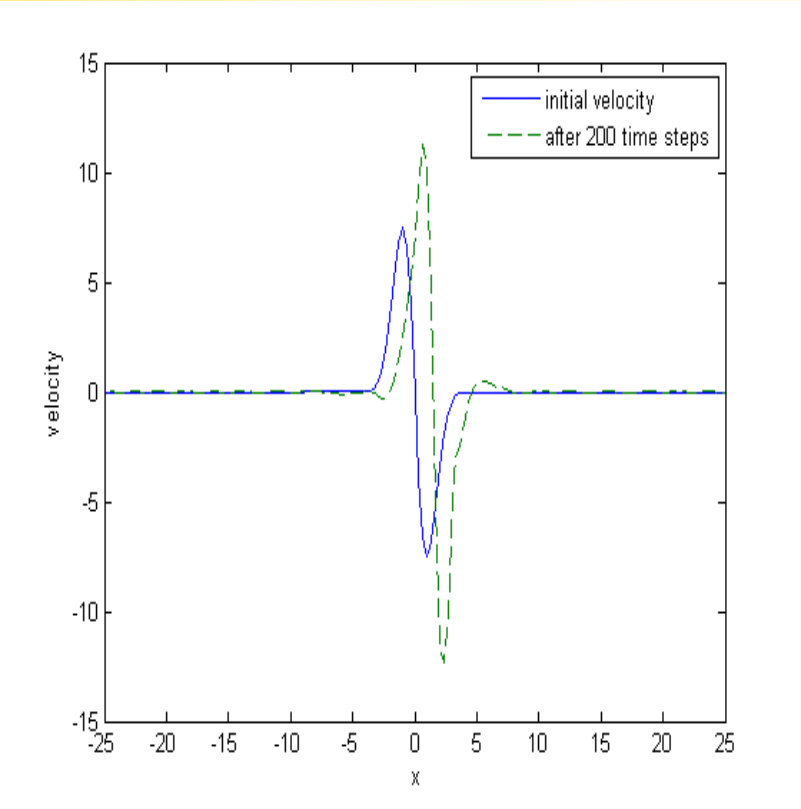




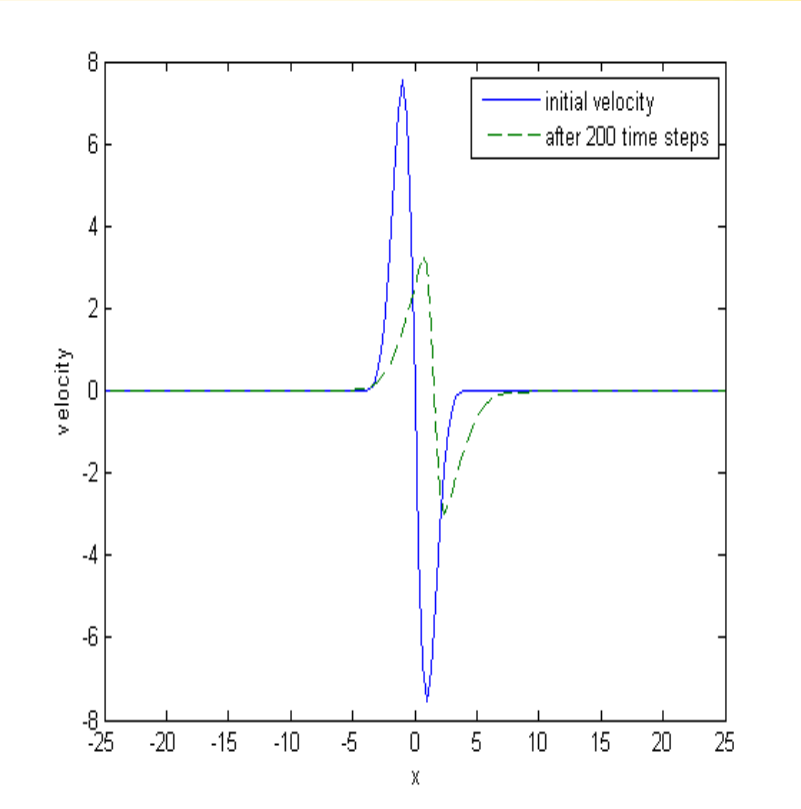
For the first-order upwind scheme the threshold value of vorticity is used,:

$$\omega > 0.1\omega_{max}$$

Taylor vortex convection (Ma=0.25) using AVC with a) second-order scheme and b) first-order scheme

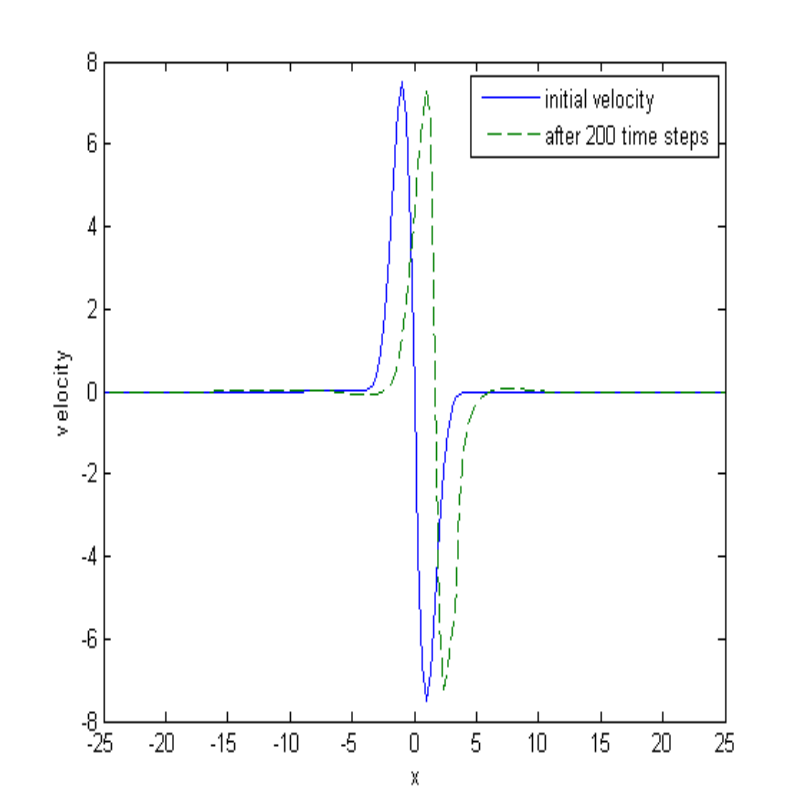


second-order scheme

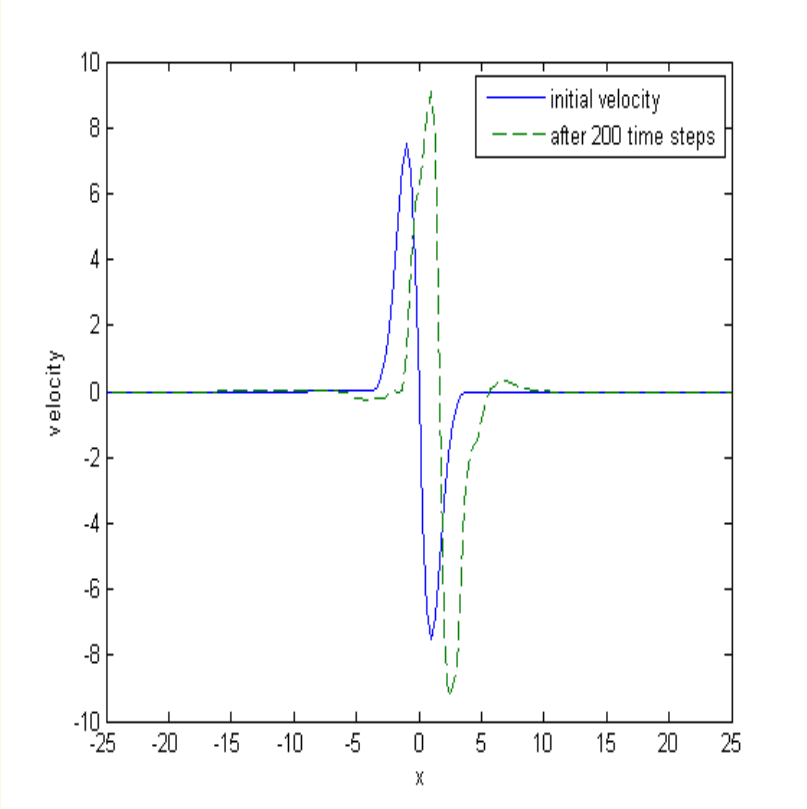


first-order scheme

Convected Taylor vortex with different AVC threshold values: a) 10% and b) 1%.

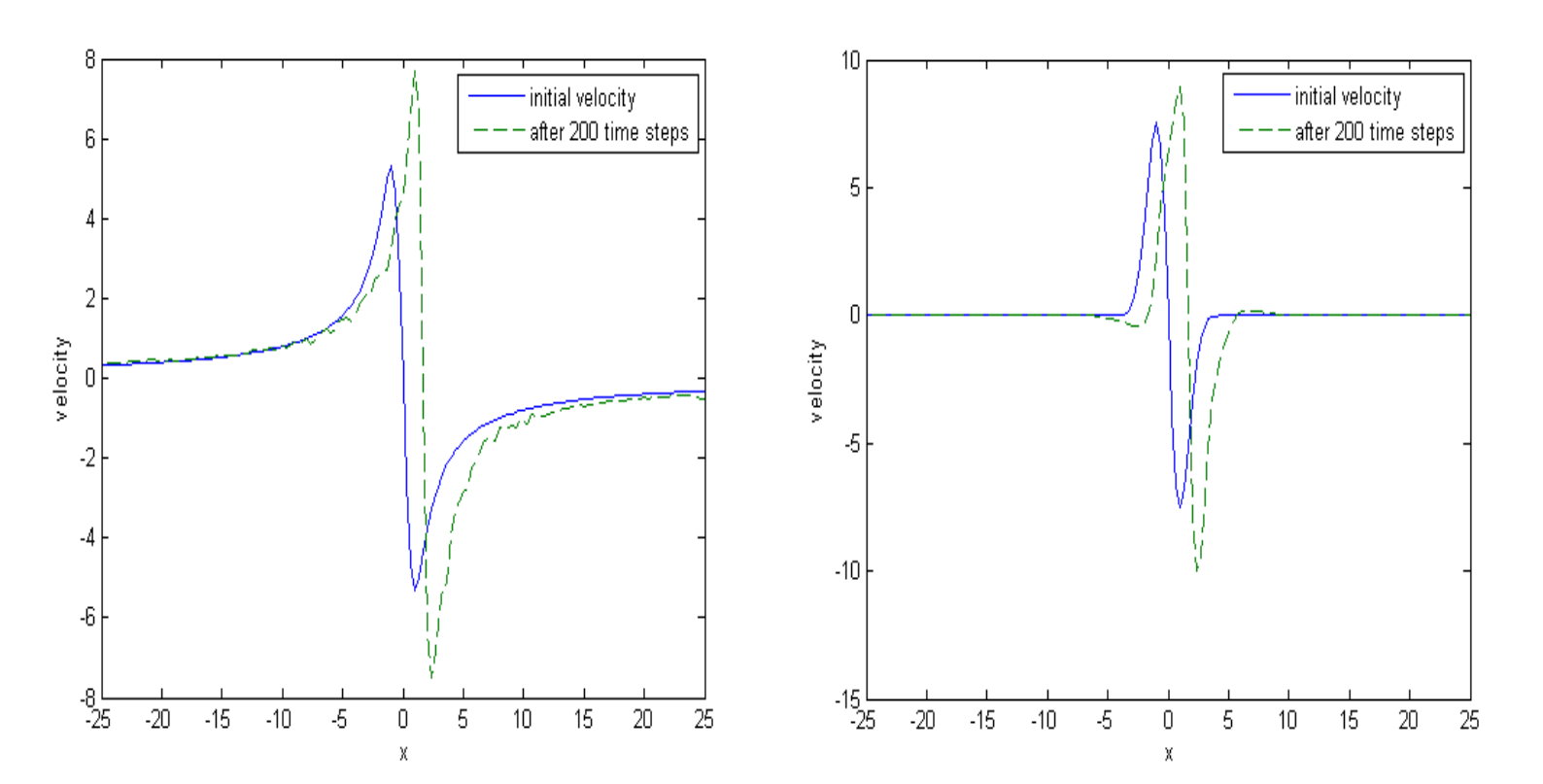


a) 10%,



b) 1%

Convected (Ma=0.25) Vatistas (a) and Taylor (b) vortices. Second- order TVD scheme is utilized. Value of AVC threshold is equal to 0.



Vatistas vortex

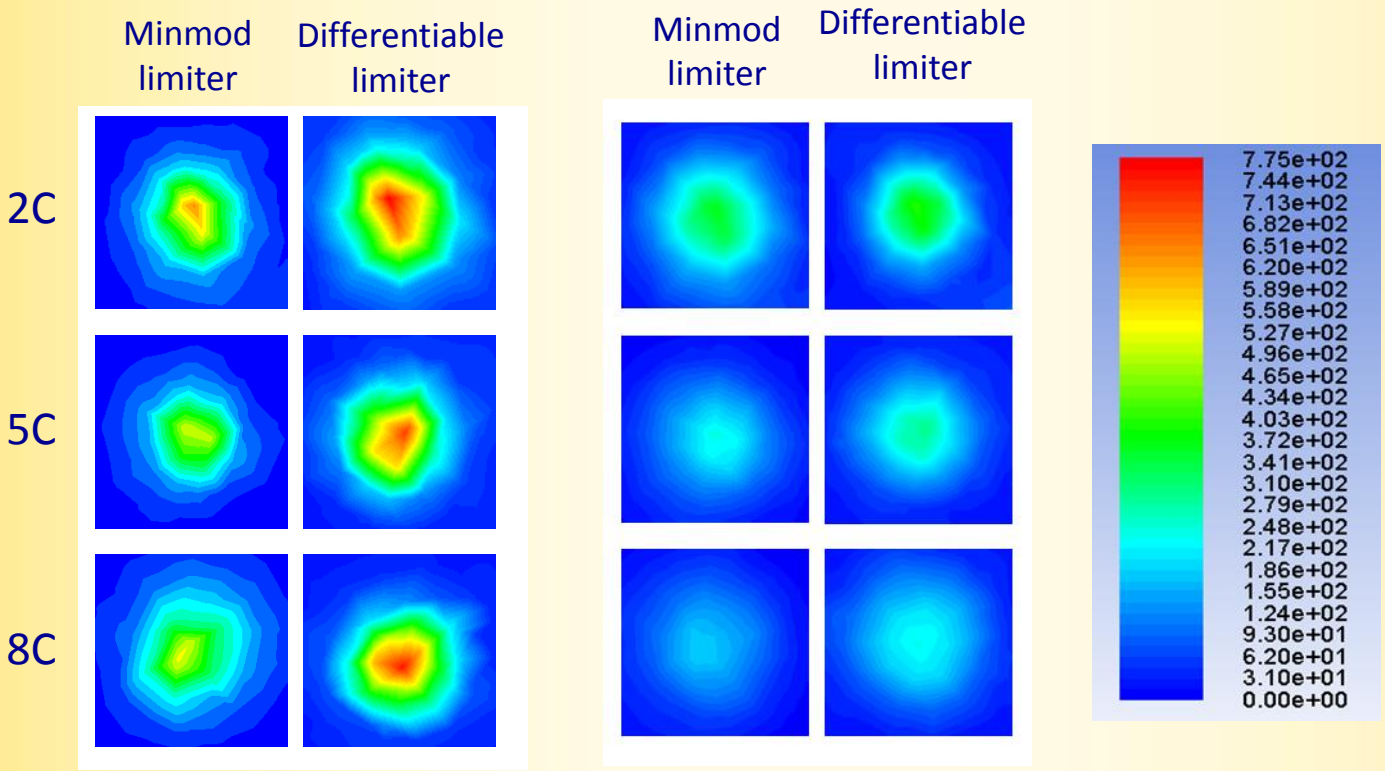
Taylor vortex

3D Fixed Wing, Inviscid Model Settings

FLUENT Settings	
Solver and Models	
Solver	Pressure Based
Viscous Model	Inviscid
Density	Ideal Gas
Energy Equation	On
Solution Methods	
Pressure-Velocity Coupling	SIMPLE
Density, Momentum, Energy	Second Order Upwind
Pressure	Standard

Differentiable and Minmod TVD Limiters

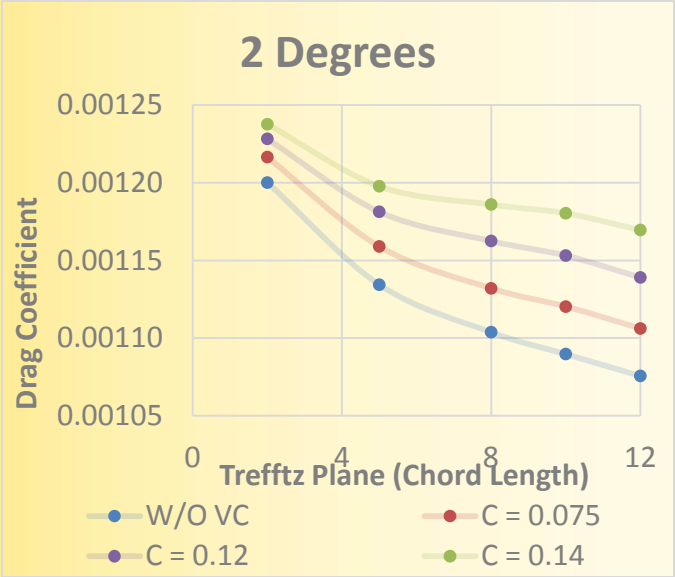
- The differentiable limiter captured a higher level of vorticity compared to the minmod limiter; for the Minmod limiter, c=0.075; for the differentiable limiter, c=0.0525

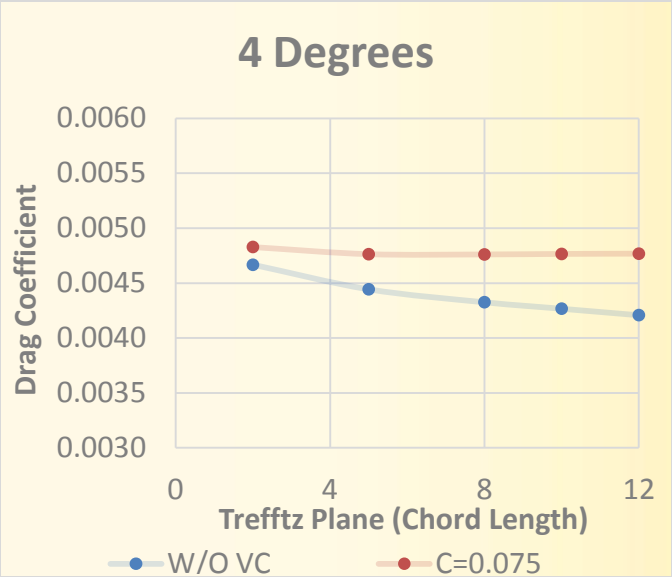


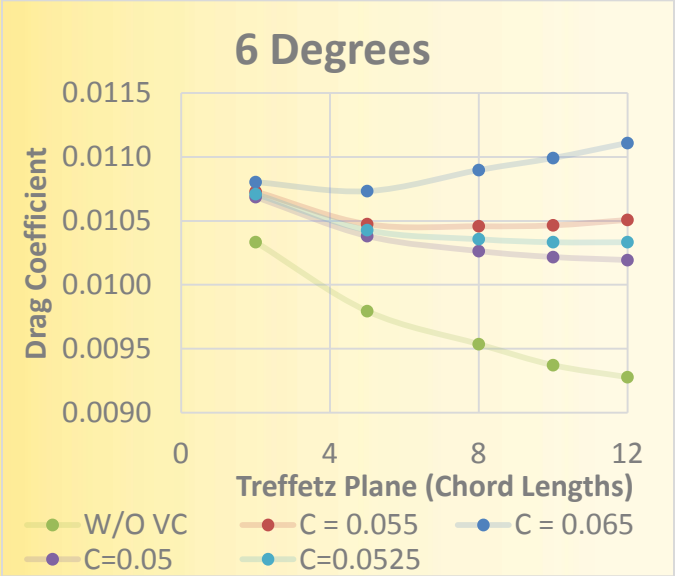
With VC

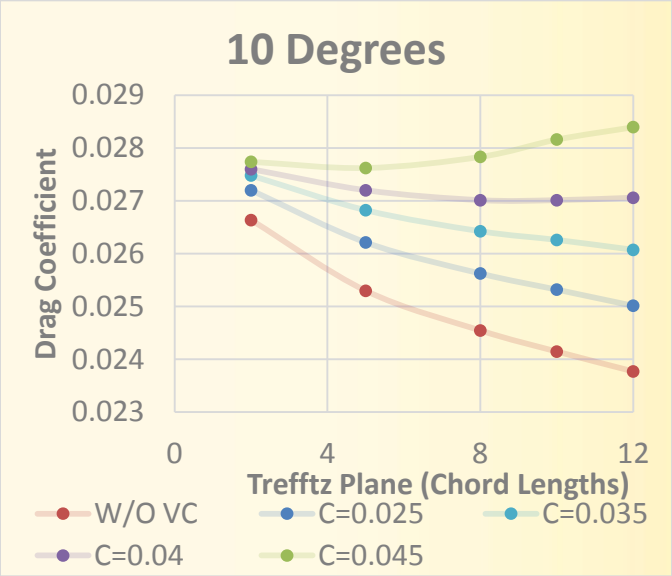
Without VC

2, 4, 6 and 10 Degrees AoA
Ma = 0.3









Comparison to lifting line theory

Drag Coefficient					
M=0.3					
AoA	Lifting Line Theory	Surface Int.	Wake-Int. with VC	Surf. Int. Error %	Wake-Int. Error %
2	0.00136	0.0048	0.00118	253	13
4	0.00544	0.0116	0.0043	113	21
6	0.0122	0.021	0.0104	72	15
10	0.034	0.052	0.027	53	-21
M=0.5					
2	0.0018	0.0055	0.00133	206	26
4	0.0072	0.013	0.0055	81	24
6	0.016	0.0238	0.0121	49	24
M=0.6					
2	0.0021	0.0143	0.00153	581	27
4	0.0084	0.0143	0.0061	70	27

Confinement Parameters			
AoA	M=0.3	M=0.5	M=0.6
2	0.14	0.14	0.14
4	0.075	0.07	0.065
6	0.0525	0.05	
10	0.04		

Viscous decay of line vortex

- To extend to viscous/turbulent simulations, the effect of VC on the decay of a line vortex was studied.

- The initial velocity distribution at t=0 is given by:

$$U = \frac{\Gamma}{2\pi r}$$

- The decay of the tangential velocity, U, by viscous forces can be determined analytically by:

$$U = \frac{\Gamma}{2\pi r}\left(1 - e^{-r^2/4vt}\right)$$

Scales relevant to tip vortex dynamics

- The maximum value of effective kinematic viscosity $\nu_{eff} = \frac{\mu_{eff}}{\rho}$ per cross-section:

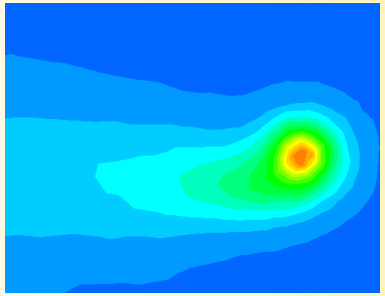
  $\nu_{eff}<10^{-2}$

- The residence time of convected fluid particle from wing tip to Trefftz plane is:

10 Chords @ 1m/chord

$t=\frac{10\ m}{100\ {}^{m}/_{s}}=0.1$

$\nu t<10^{-3}$



Effective viscosity contours between 0.001 and 0.0025

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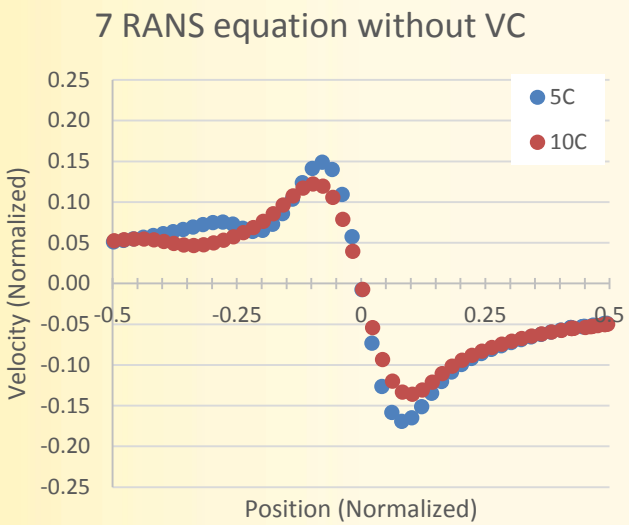
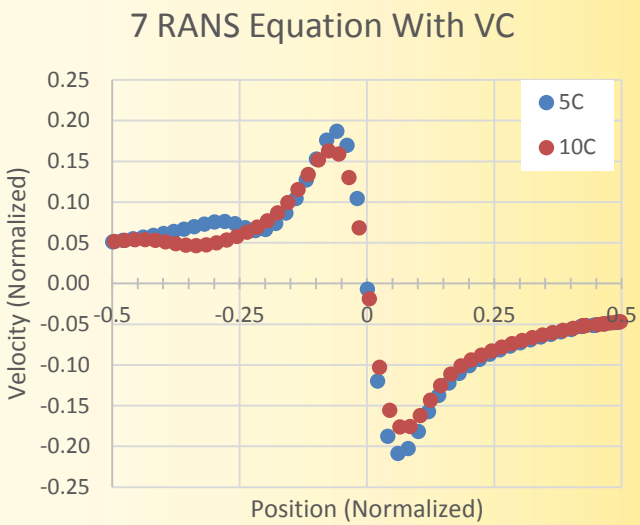
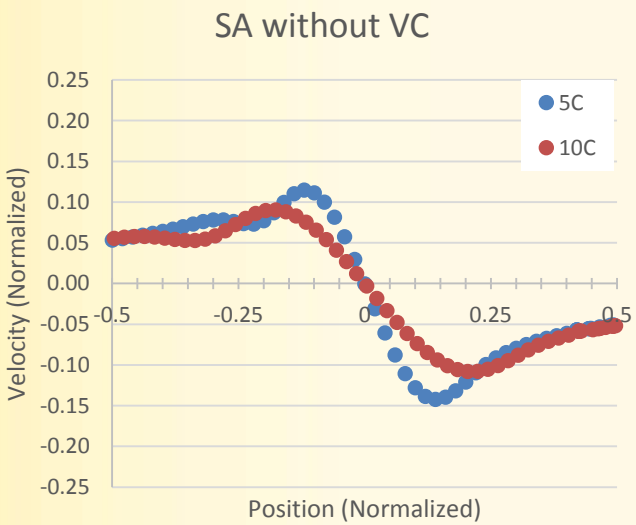
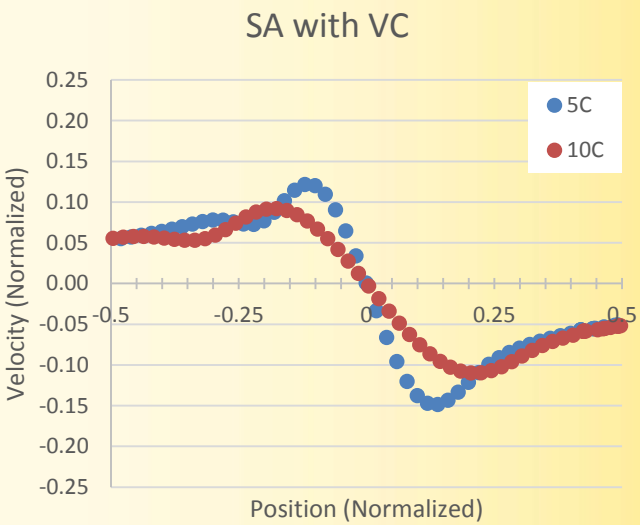
Results: Model of turbulence and VC

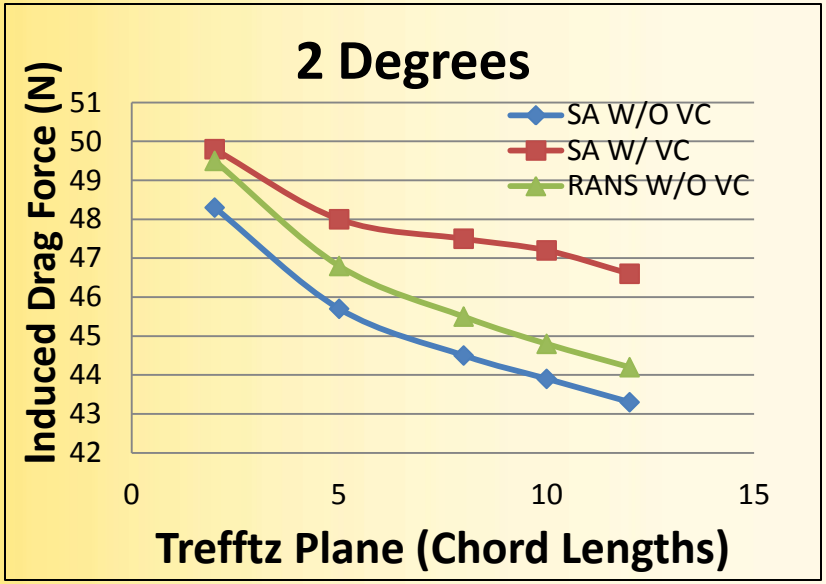


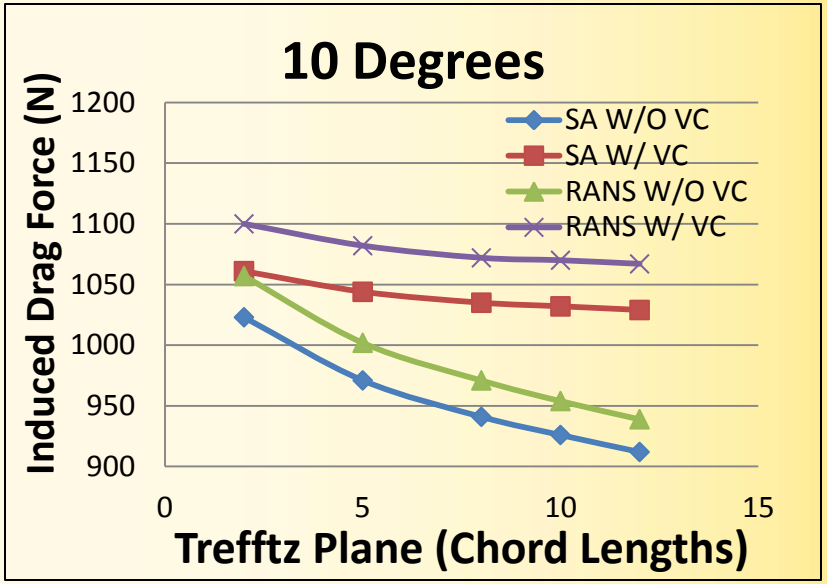


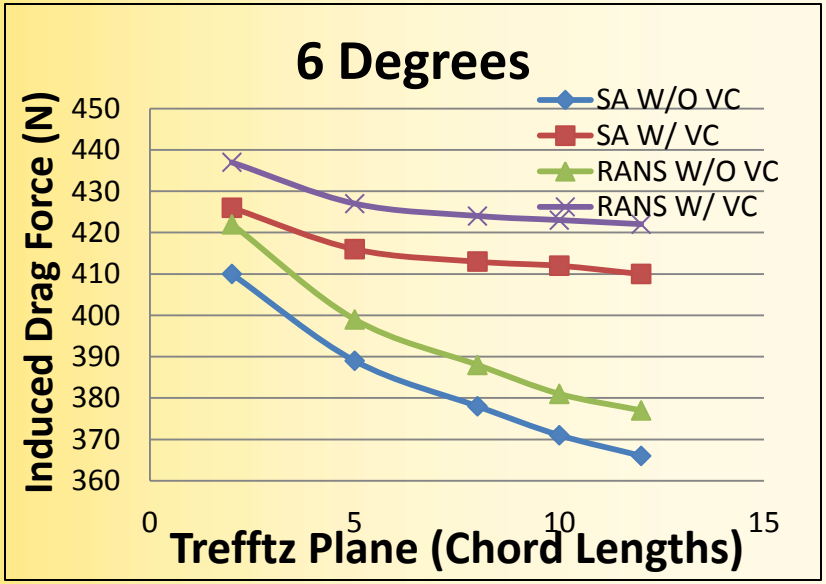




Ma=0.3, Turbulent models







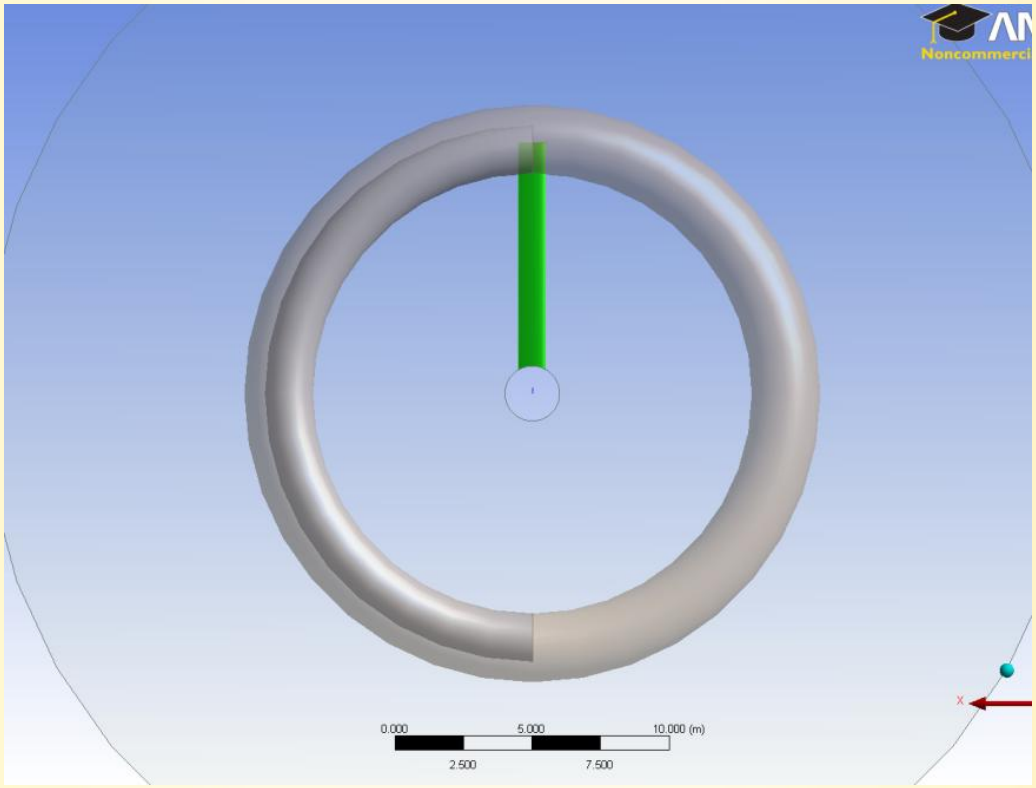
Shear Force (N)	
2 Degrees	
Spalart-Allmaras	301
RANS	159
6 Degrees	
Spalart-Allmaras	282
RANS	150
10 Degrees	
Spalart-Allmaras	255
RANS	145

Application of VC to Hovering Rotor

Problem Specifications	
Wing Profile	NACA 2415
Aspect Ratio	9.12
Tip Mach Number	0.26
Pitch	4.5 Degrees

FLUENT Settings	
Solver and Models	
Solver	Pressure Based
Viscous Model	Inviscid
Density	Ideal Gas
Energy Equation	On
Solution Methods	
Pressure-Velocity Coupling	SIMPLE
Density, Momentum, Energy	Second Order Upwind
Pressure	Standard
Confinement Parameter	0.04

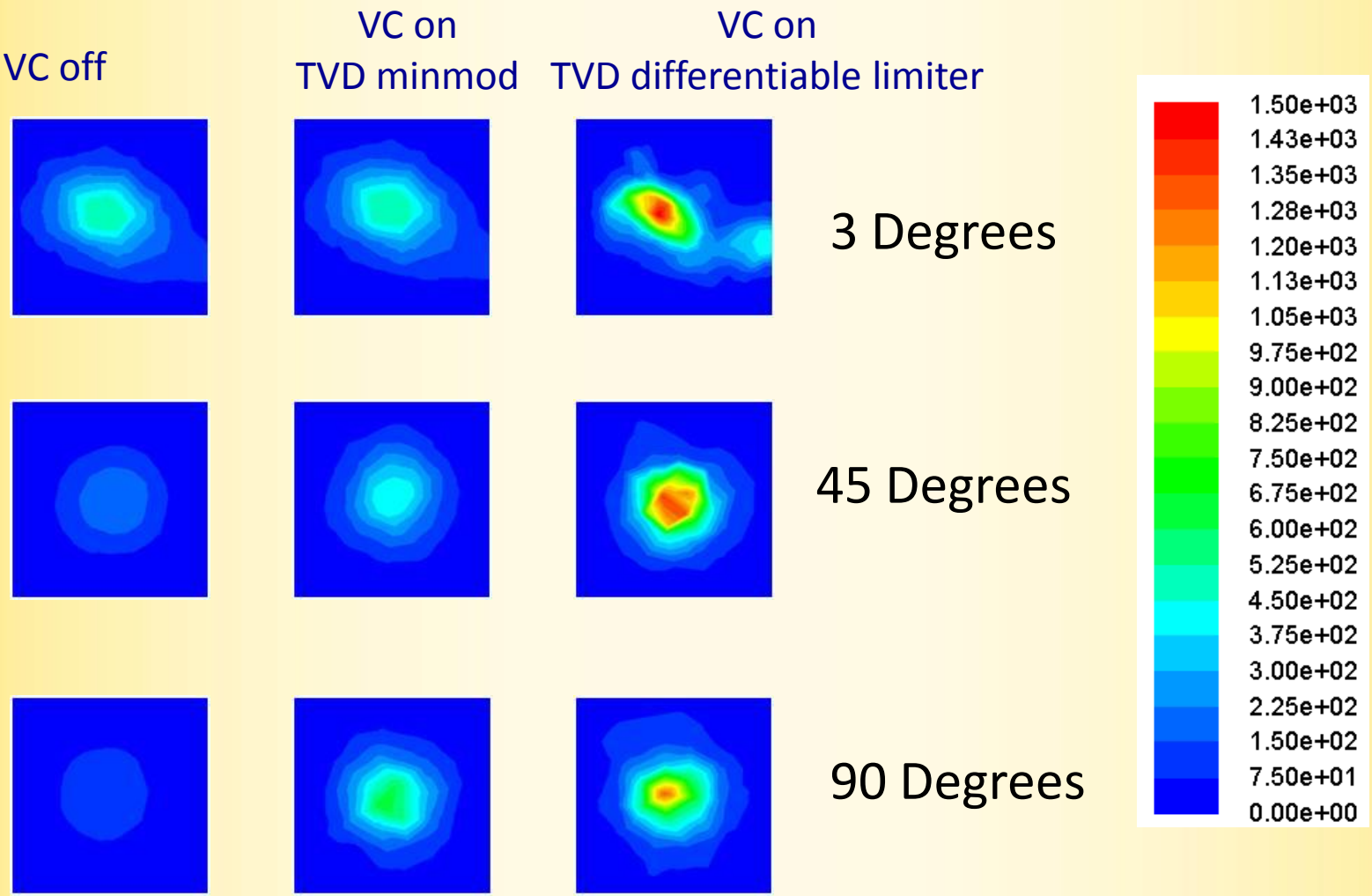
Hovering Rotor Grid



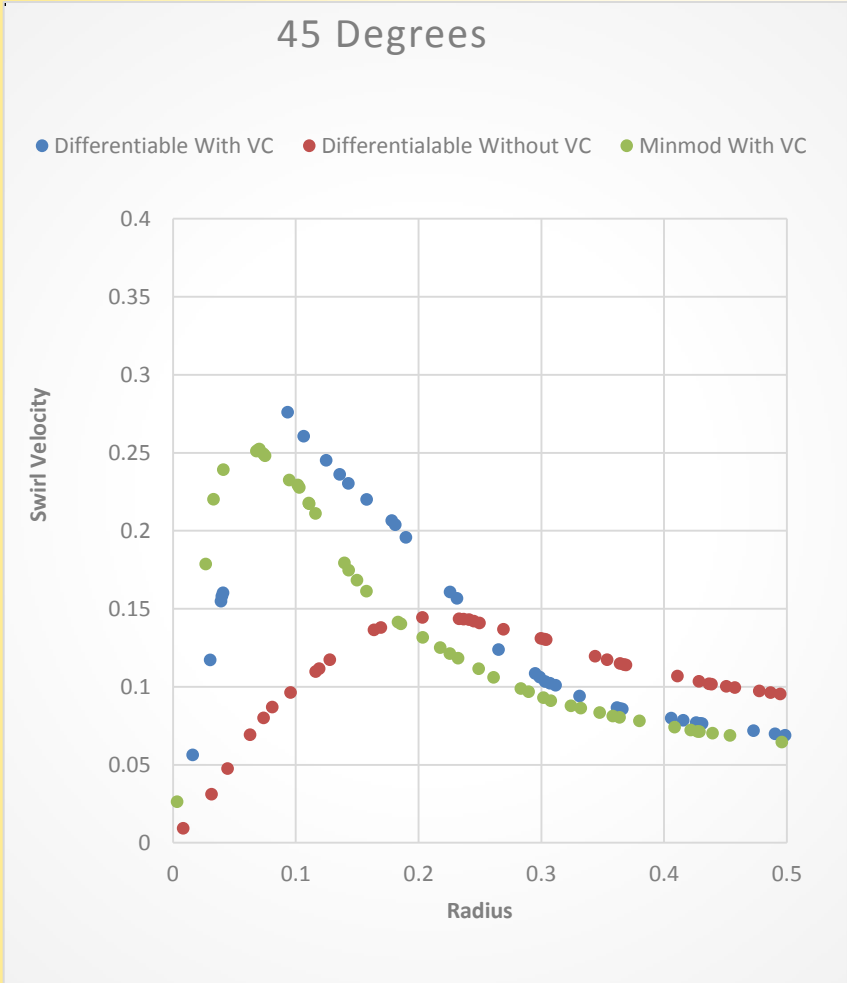
Total number of cells: 5,841,659 cells

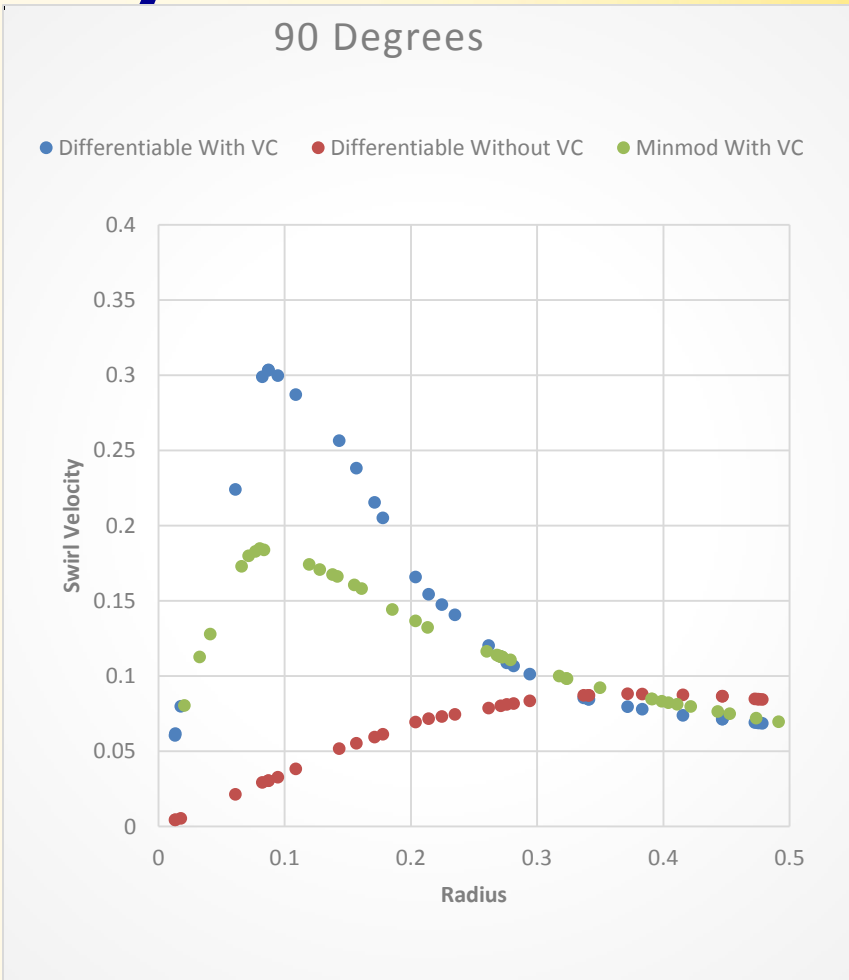
Experiment: Duraisamy, K., Baeder, J.D, “High Resolution Wake Capturing Methodology for Hovering Rotors,” Journal of the American Helicopter Society 52(2), 2006, pp. 110 -122.

Effect of VC with TVD minmod and differentiable limiters



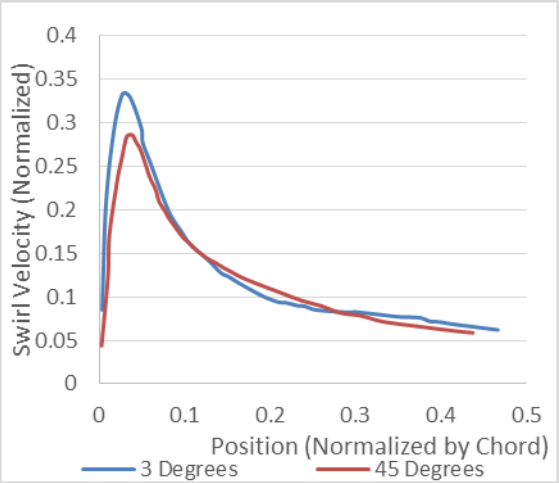
Inviscid modeling: Comparison of *minmod* and differentiable limiters. Rotational velocity in vortex:

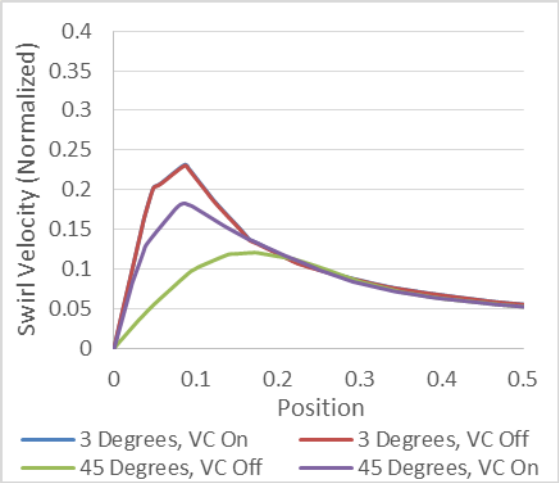


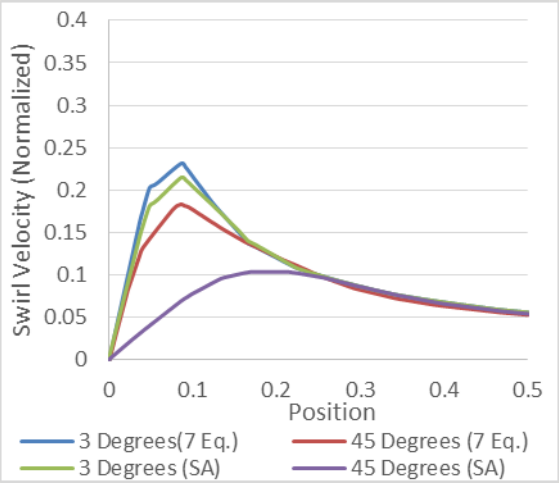


Experimental data: Normalized velocity has the value of 0.35

Swirl turbulent velocity of wingtip vortex







Experiment:
Duraisamy, K., Baeder, J.D
Journal of the American Helicopter Society
52(2), 2006, pp. 110 -122.

Six Reynolds Stresses RANS model with and without VC

Six Reynolds Stresses vs Spalart-Allmaras Turbulence model with VC enabled

VC/TVD and RANS model of turbulence for rotating blade with winglet

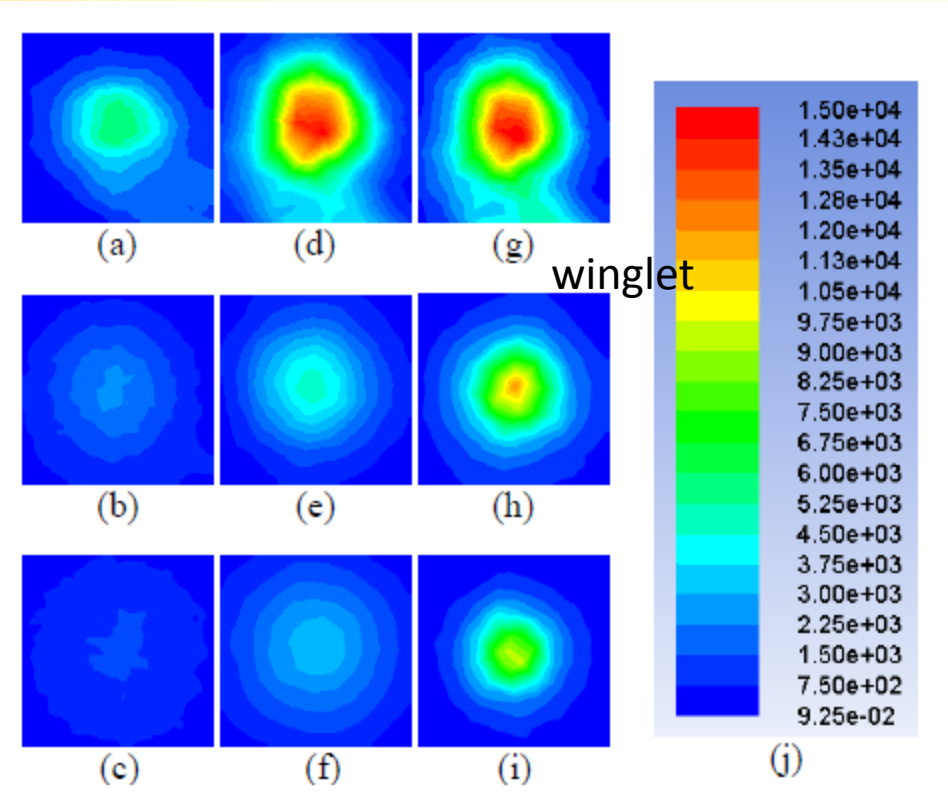
Upwind discretization

1st 2nd VC

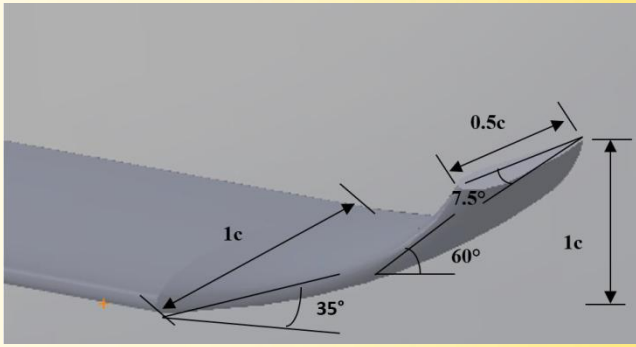
3 degrees

45 degrees

90 degs



winglet



winglet

The same value of c is used as for airflow without winglet

Conclusions

- For tip vortex shed by rotating blade or wing, upwind schemes (even 2nd order!) create significant numerical dissipation of vortex.
- Vorticity Confinement (VC) is able to prevent the vortices from smearing till they travel angular distance of up to 90-180 degrees or linear distance of up to 15 chords.
- VC was combined with second-order TVD schemes using *minmod* and differentiable limiters.
- Comparison of TVD limiters show that differentiable van Albada limiter provides with the most accurate results.
- The proposed combination of VC with TVD cures the numerical over-confinement of vortex but some adjustment of VC parameter is needed.

Conclusion: adjusted formulation of VC (AVC) combined with TVD

- For adjusted formulation of VC (AVC) no VC parameter is needed, however, the threshold value of vorticity is required to avoid instability.
- The solution dependency on the confinement parameters is mitigated in two ways: 1) reducing their sensitivity (VC+TVD) and 2) computing them adaptively to solution (AVC) by relating the VC parameter ε to the numerical dissipation of the discretization scheme.
- TVD combined with AVC leads to stable solution and heuristic threshold value of vorticity is no longer needed.

Related Publications

1. K. Pierson and A. Povitsky, Vorticity Confinement for Turbulent Wing Tip Vortices and Wake-integral Aircraft Drag Prediction, 21st AIAA Computational Fluid Dynamics conference, AIAA Paper 2013-2574.
2. K. Pierson and A. Povitsky, Preservation of Tip Vortices of Helicopter Blades by Vorticity Confinement, AIAA Paper 2013-2420, the 31st AIAA Applied Aerodynamics conference, San Diego, CA, June 2013.
3. T. Snyder and A. Povitsky, Drag Prediction with Vorticity Confinement, 40th AIAA Fluid Dynamics Conference, AIAA Paper 2010-4935.
4. T.Snyder and A.Povitsky, Far-field Induced Drag Prediction Using Vorticity Confinement Technique, AIAA J. of Aircraft, early access, April 2014.
5. Bono Wasistho, (PI, IllinoisRocstar) , Alex Povitsky (University consultant), Mark Brandyberry, NAVY SBIR PHASE I FINAL TECHNICAL REPORT Contract Number N68335-14-C-0029 Development of Adaptive Vorticity Confinement Based CFD Methodology for Rotorcraft Applications , April 2014.

Aknowledgements:

- DAGSI grant in 2009-2012 (Sponsor Dr. J. Camberos, US Air Force Research Laboratory (AFRL))
- US Army Research Office (ARO) grant in 2012-2013 (ARO program director Dr. F. Ferguson)
- NAVY SBIR: Development of Adaptive Vorticity Confinement Based CFD Methodology for Rotorcraft Applications (Phase 1), IllinoisRocstar LLC., 2013-2014. PI: B. Vatistho, co-PI A. Povitsky
- NSF ICORP 2014
- ASEE/AFRL summer faculty and student award (AFRL/WPAFB, Dayton, OH) in 2011 and 2012

- Supported graduate students:

Kris Pierson (graduated with M.Sc. in 2014)

Troy Snyder (graduated with M.Sc. in 2012)

Questions

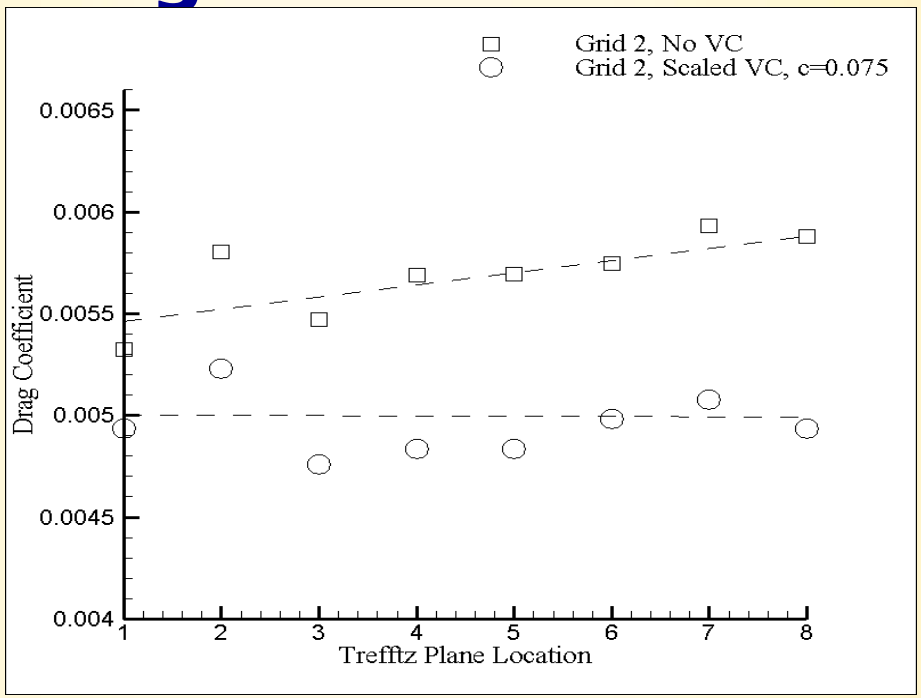
?

Differential limiter:

- Z. J. Wang.
  A Fast Nested Multi-grid Viscous Flow Solver for Adaptive Cartesian/Quad Grids.
  International Journal for Numerical Methods in Fluids, 33:657-680, 2000.

- V. Venkatakrishnan.
  On The Accuracy of Limiters and Convergence to Steady State Solutions.
  AIAA-93-0880, American Institute of Aeronautics and Astronautics, January 1993.

Numerical entropy drag coefficient using inviscid grid:



We show how the VC controls the generation of spurious entropy in the wake. In Figure , the behavior of the wake-integral entropy drag coefficient is depicted with and without the application of VC. Without VC, the entropy drag coefficient generally increases as the Trefftz plane position moves farther downstream. This is indicative of the transfer from induced to entropy drag accompanying the numerical dissipation of the wing's trailing vortices.

The VC preserves the trailing vortices and helps counteract such behavior.

Viscous/turbulent/shock wave cases: Integration in Trefftz plane should include integration of entropy.

- The wake-integral method can provide insight into the physical sources of drag since the drag can be decomposed into its relevant components, associated with physical phenomena (viscous drag, shock wave drag, and induced drag), as separate integrals.

See R. M. Cummings, M. B. Giles, and G. N. Shrinivas, "Analysis of the elements of drag in three-dimensional viscous and inviscid flows," in AIAA-96-2482-CP, AIAA, 1996.

- In the near future, the combined VC method, wake-integration technique and entropy cut-off will be applied to flows with shock waves and boundary layers. The separation of scales of numerical and physical entropy by applying VC will be used to refine the entropy cut-off technique to suppress spurious entropy drag and reduce computational overhead.

Stationary Wing and Grid

- A NACA0012 airfoil profile was used to construct a 3D wing with an aspect ratio of 6.67.
- The computational domain consisted of 1.38 million cells and extended 14 chord lengths away from the wing.
- The region of refinement was designated from the tips of the wing to the edge of the computational domain; this is the area that the tip vortex tube is expected to be.

Wake-Integral Method

- Applies a control volume over the aerodynamic body
- Under 1-D flow assumptions at exit cross-section, this method can be simplified to include only integration over a cross-flow plane within the wake (known as the Trefftz Plane).
- $F_D = -\left[\oint P_{out} dS - \oint P_{in} dS + \oint \rho u^2 dS - \oint \rho_\infty U_\infty^2 dS\right]$
- The induced component of drag can be determined by $F_{D_i} = \int_{wake} \frac{\rho_\infty}{2}(v^2 + w^2) dy dz$

Scales relevant to tip vortex dynamics

The maximum value of effective kinematic viscosity $\nu_{eff} = \frac{\mu_{eff}}{\rho}$ per cross-section

$$\nu_{eff} < 10^{-2}$$

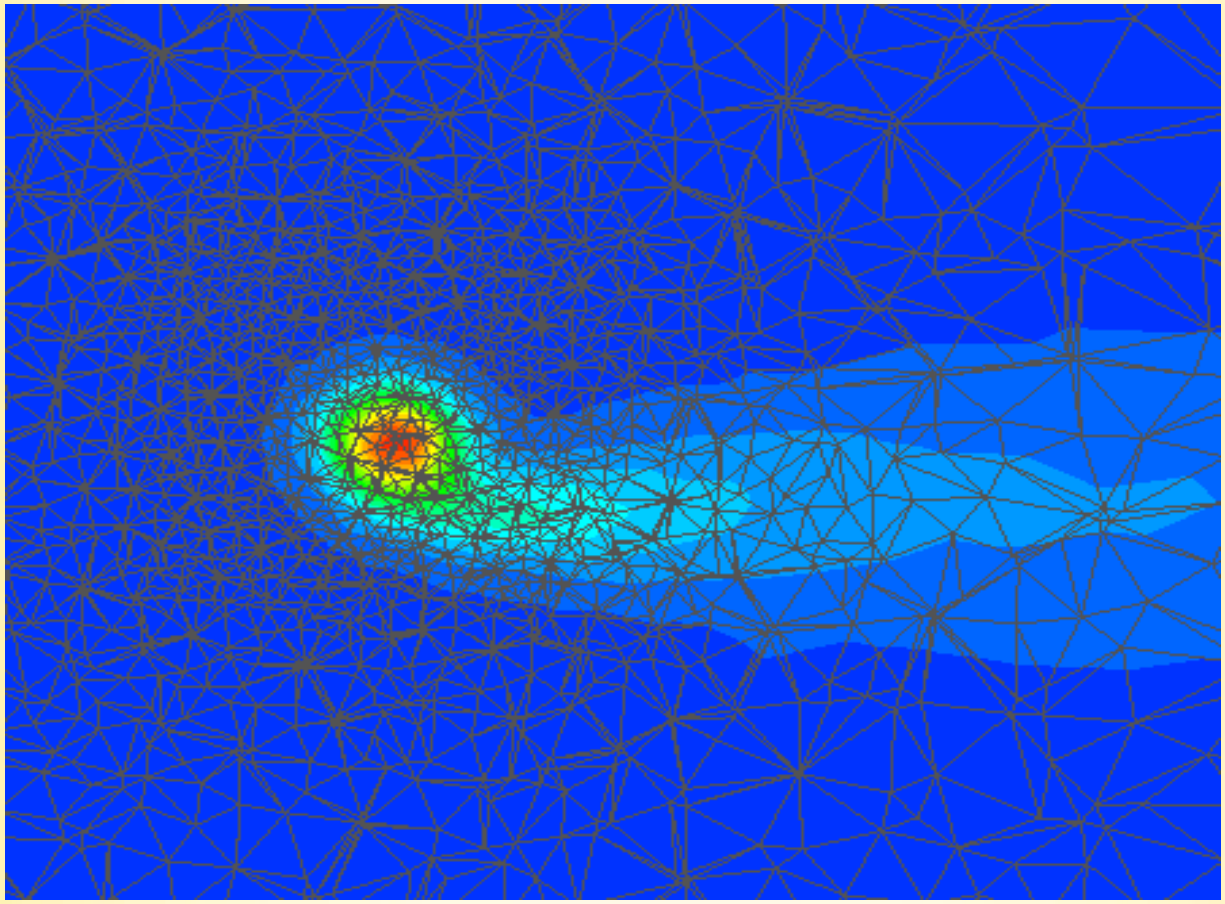
The residence time of convected fluid particle from wing tip to Trefftz plane is

10 Chords @ 1m/chord

t=10m/100 m/s=0.1

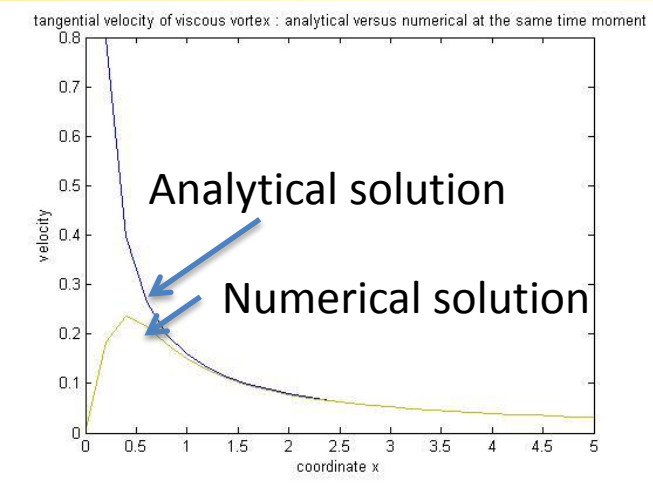
$$\underline{\nu t < 10^{-3}}$$

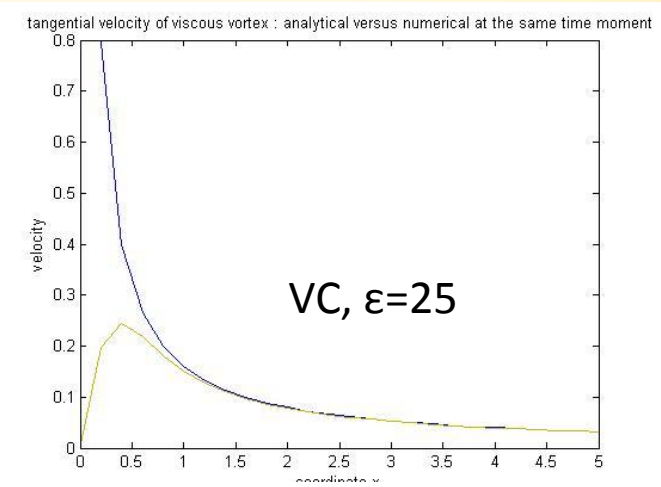
Estimate of effective viscosity for turbulent tip vortices

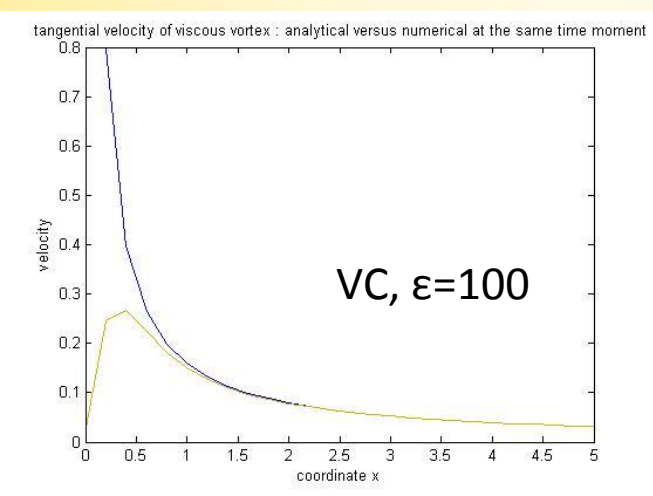


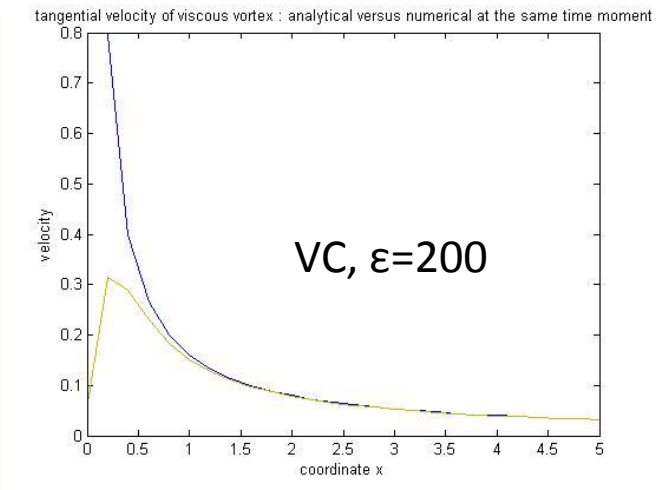
Without VC(20 Contours of effective viscosity between 0.001 and 0.0035)
2 chord lengths downstream

Decay of line vortex time x viscosity (vt) =0.001 (200 time steps)









Grid step

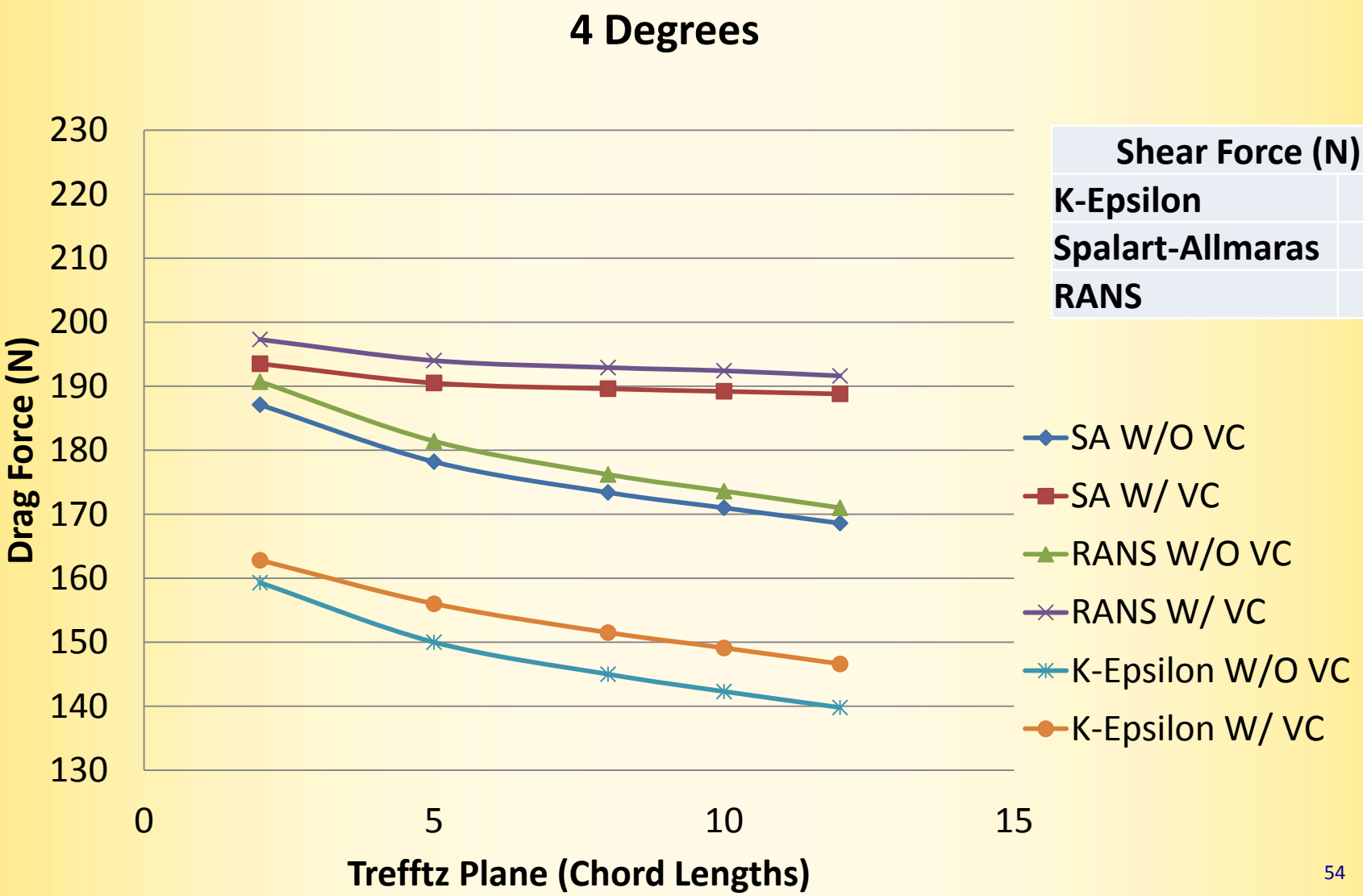
$$\Delta_x = \Delta_y = 0.1$$

The effect of confinement for this range of ε minor. Note that ε =200 is a case of over-confinement.

3-D Turbulent tip vortices

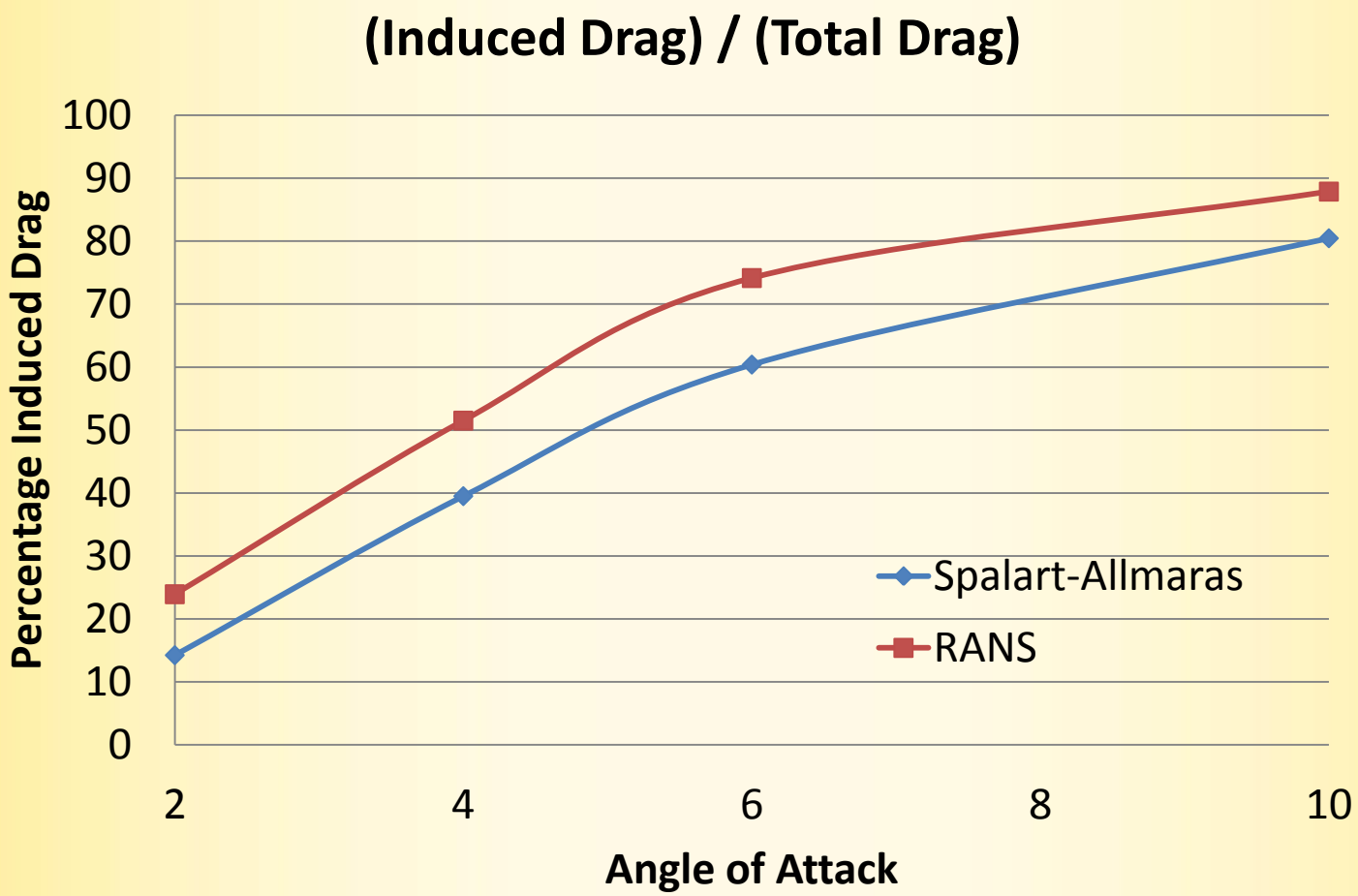
- Estimate the effect of VC on turbulent dissipation in tip vortices
- Show that a properly tuned VC confinement parameter will not effect viscous dissipation
- Determine most applicable turbulence models for these simulations
  - K-Epsilon, Spalart-Allmaras, RANS models tested
- VC settings are the same as in inviscid cases

4 Degrees Turbulent



Shear Force (N)	
K-Epsilon	395
Spalart-Allmaras	291
RANS	179

Percentage Induced Drag in Total Drag



Taylor and Vatistas vortex velocity profiles:

Taylor profile:

$$u = -y\exp\left(0.5\left(1-(x^2+y^2)\right)\right), v = x\exp\left(0.5\left(1-(x^2+y^2)\right)\right)$$

Vatistas profile:

$$u = -y/(1+(x^2+y^2)^n)^{1/n}, v = x/(1+(x^2+y^2)^n)^{1/n},$$

See Larry Young, Vortex Core Size in the Rotor Near-Wake, NASA /TM 2

Velocity and vorticity profiles (a) Taylor vortex and (b) Vatistas

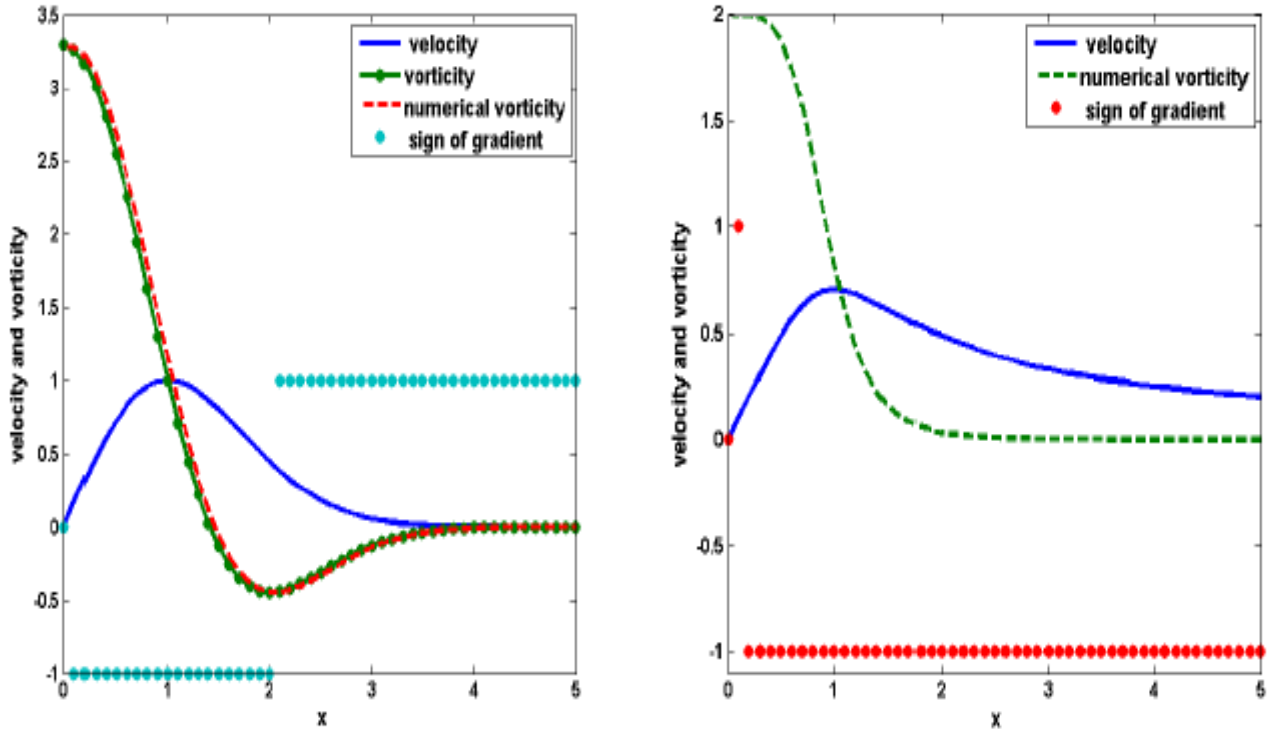


Figure 3.1.1: Velocity and vorticity profiles along vortex radius (x axis): (a) Taylor vortex and (b) Vatistas vortex.

2DFlow with Vorticity Confinement

- Specifics of solver
  - Compressible Euler/Navier-Stokes
  - Structured finite-volume
  - Steger-Warming flux splitting
  - Upwind discretization of convection terms
  - Central discretization of viscous terms
  - First-order explicit time stepping
- VC body force
  - Vorticity and vorticity gradient are computed by central scheme

IMPLEMENTATION OF AVC IN CFD code *RocfloMP*

The vorticity confinement methods have been implemented in high-fidelity block structured CFD code *RocfloMP*

RocfloMP employs a cell-centered, finite-volume spatial discretization and utilizes the Arbitrary Lagrangian-Eulerian (ALE) method for dynamic and/or moving grids.

http://www.csar.illinois.edu/CSARdocs/DocumentsPub/uguides/rocflo_ug.pdf

See more details:
NAVY SBIR PHASE I FINAL TECHNICAL REPORT Contract Number N68335-14-C-0029
Development of Adaptive Vorticity Confinement Based CFD Methodology for Rotorcraft Applications
Authors: Bono Wasistho, (PI, IllinoisRocstar) Alex Povitsky (University consultant), Mark Brandyberry