M4D
AN OPEN SOURCE RESEARCH CFD CODE FOR THE CALCULATION OF CLASSICAL AND TURBULENT FLOWS

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Presentation at NIA
August 2015

M4D - input commands -> procedure:

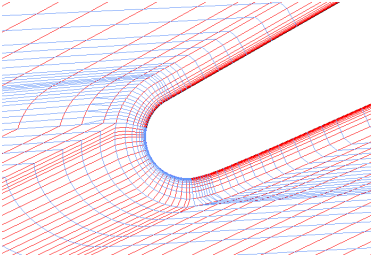
Time accurate or steady

Inviscid, laminar or turbulent

Incompressible -

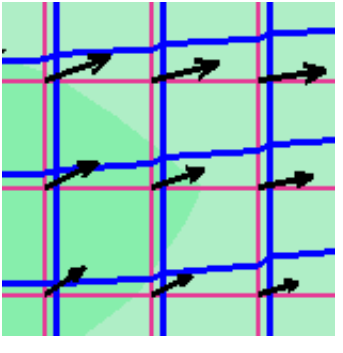
pressure correction method

Single block grid

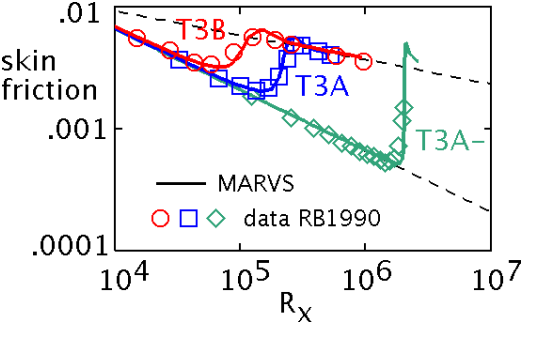


Paradigm shift features:

3-d linear profiles over convection adapted control volumes



MARVS Reynolds stress model transition - shear flow at high S/ω



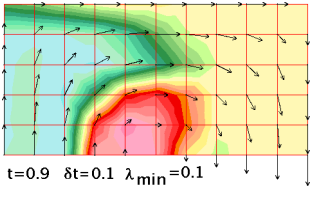
Part 1:

USING MULTI-DIMENSIONAL LINEAR DISCRETIZATION OVER UNSTEADY CONVECTION ADAPTED CONTROL VOLUMES

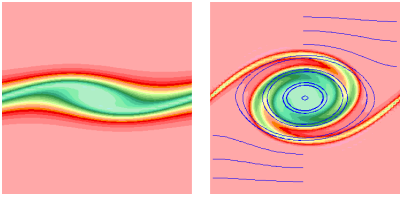
Joan G. Moore and John Moore

AIAA Paper 2014-2780

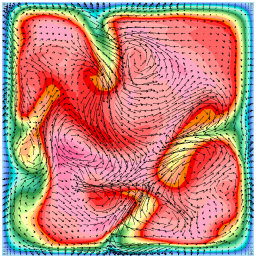
Box Cavity



Kelvin-Helmholtz Instability



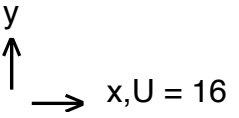
Square Channel “DNS”



Ancient History - 1970's - Steady Flow - Convection Choices

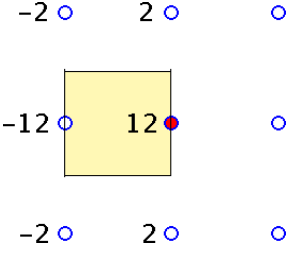
$$U_i \frac{\partial T}{\partial x_i} = 0$$

flow ➔



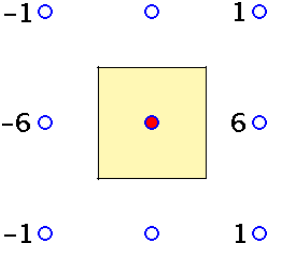
Coefficients of *T* ?

Space marching
Upstream volume
T 3-d linear
Stable - 2nd order



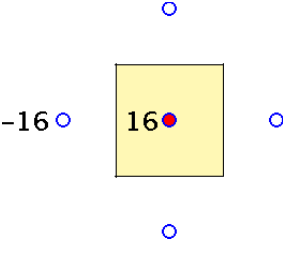
$$A_{center} \Big/ \sum A^{+} = 0.75$$

Take fully 3-D?
Centered volume
T 3-d linear
Unstable



$$A_{center} \Big/ \sum A^{+} = 0$$

Fully 3-D solution
Centered volume
T stepwise, upwind
Stable - 1st order



$$A_{center} \Big/ \sum A^{+} = 1$$

1970's fully 3-d solution introduces grid spacing, orthogonality limits and numerical mixing. Remnants remain in many of today's codes.

MEFP - 1984 - 3d steady code

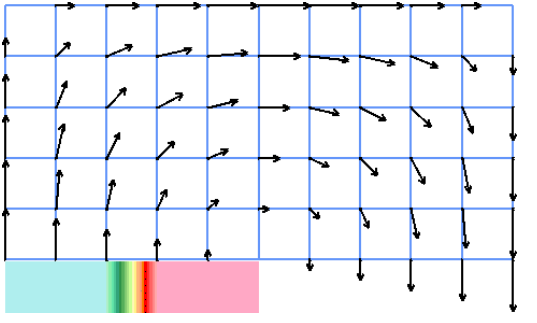
$$U_i \frac{\partial T}{\partial x_i} = 0$$

3-D linear T profiles

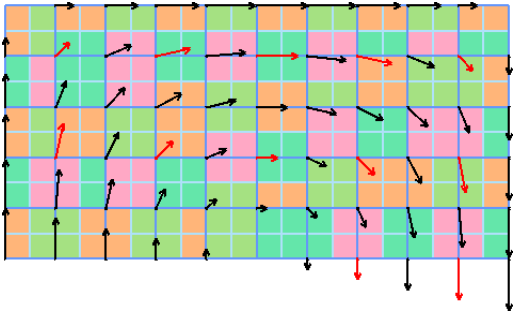
1/8th volumes are assigned to give control volumes which are upstream of the points.

Stable - 2nd order

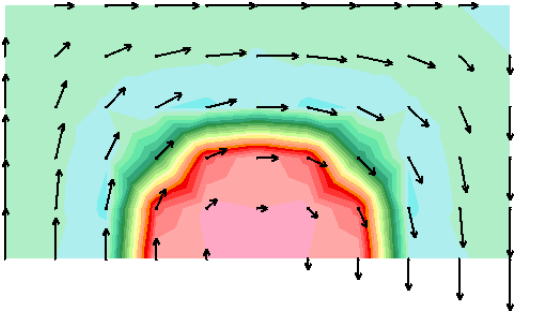
IAHR Box Cavity Convection Test
Given velocity and T-inlet



MEFP control volumes



MEFP results

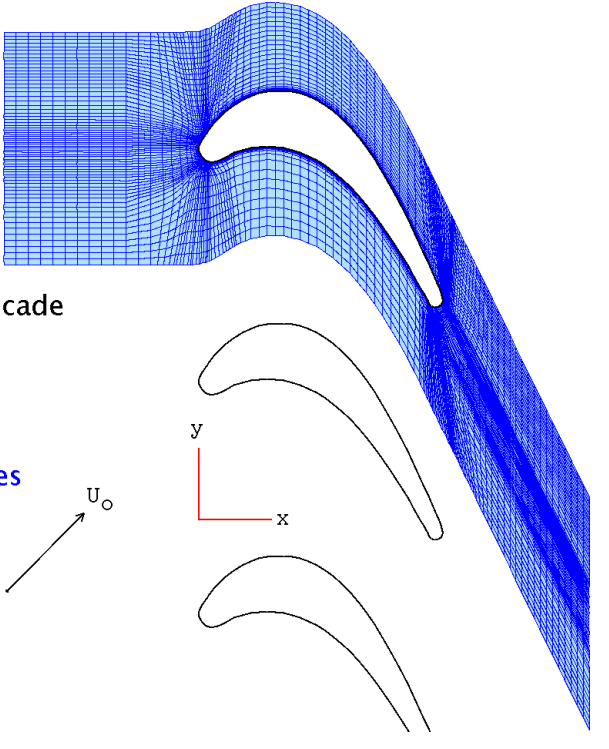


MEFP grid for turbine cascade

not orthogonal

spacing ratio mostly < 2

ok with 3-d linear profiles

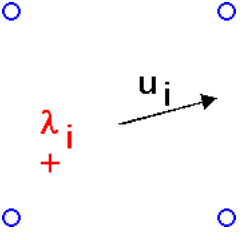


This paper - M4D - 2014 - unsteady (4d) code

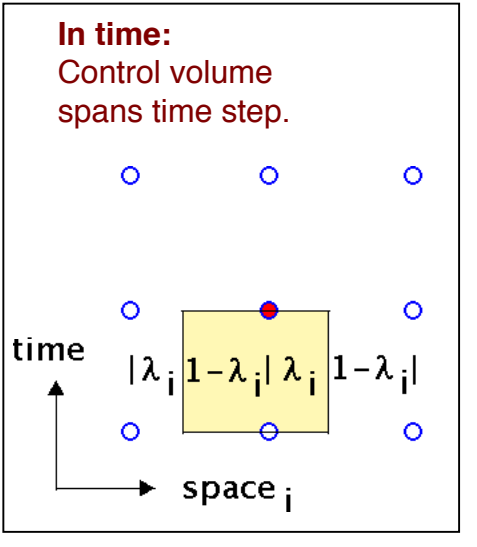
Continuous convection adapted control volumes for unsteady flow.

In space:

Control volume divide location λ_i determined
for each between-the-points volume
for each time step
based on average velocity, u_i, and viscosity.



Move divide location from center towards sides with incoming flow.



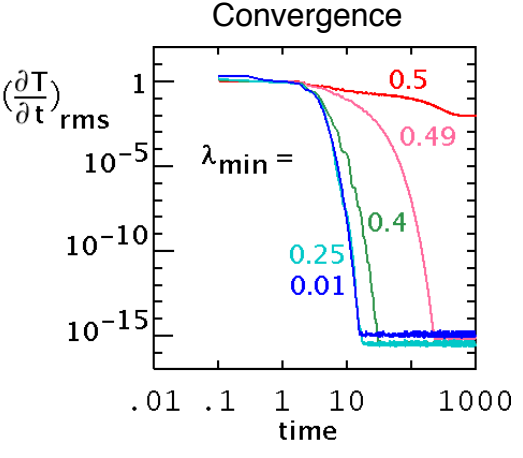
Limit λ_i: $\lambda_{\min} \le \lambda_i \le 1 - \lambda_{\min}$, where $0 < \lambda_{\min} \le 0.5$

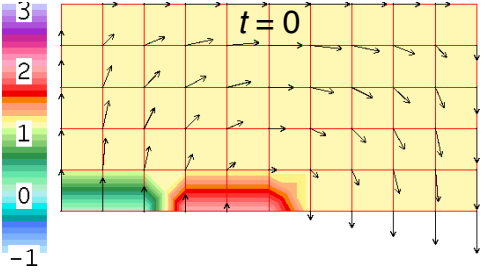
-> control volume surfaces do not coincide with grid surfaces.

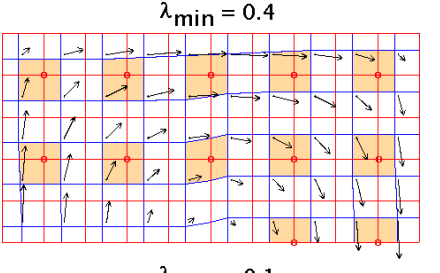
Box cavity test - unsteady transient

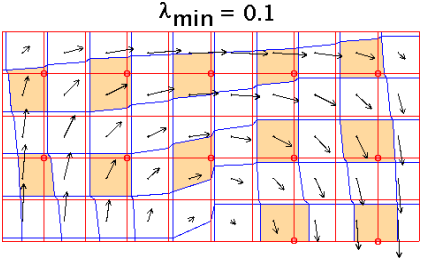
Solve $$\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = 0$$

Use: $$\delta t = \frac{\delta x}{u_{\max}} = 0.1$$

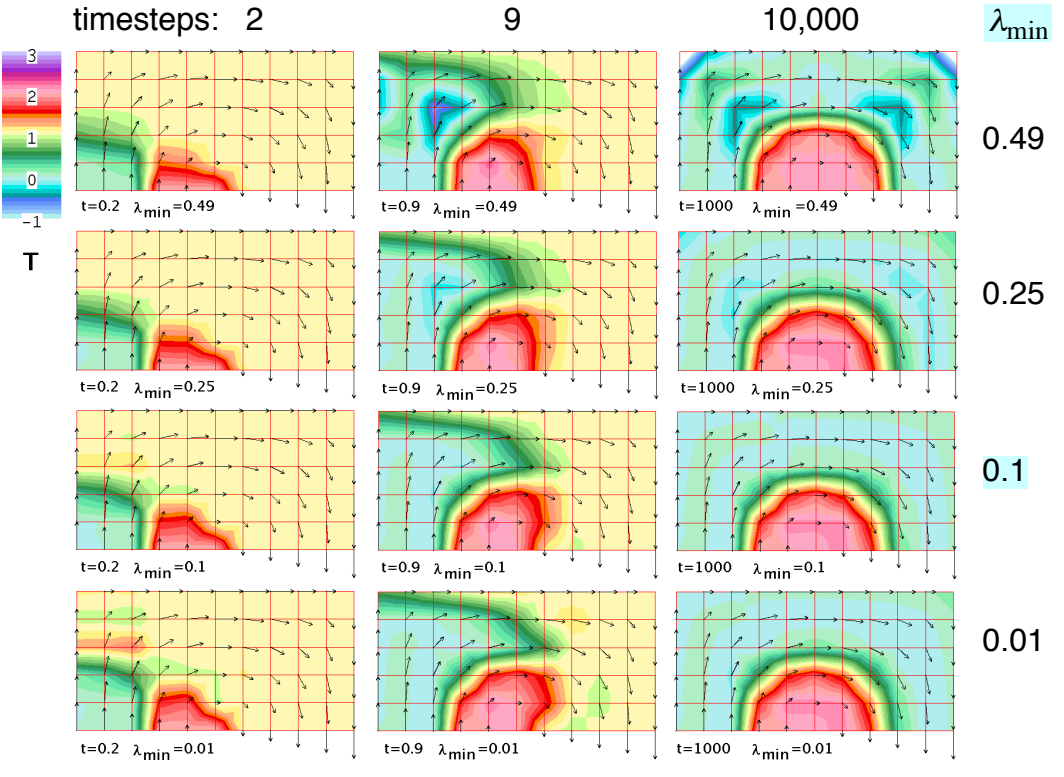








Transient solutions show dispersion/mixing errors. Best choice: $\lambda_{\min} = 0.1$



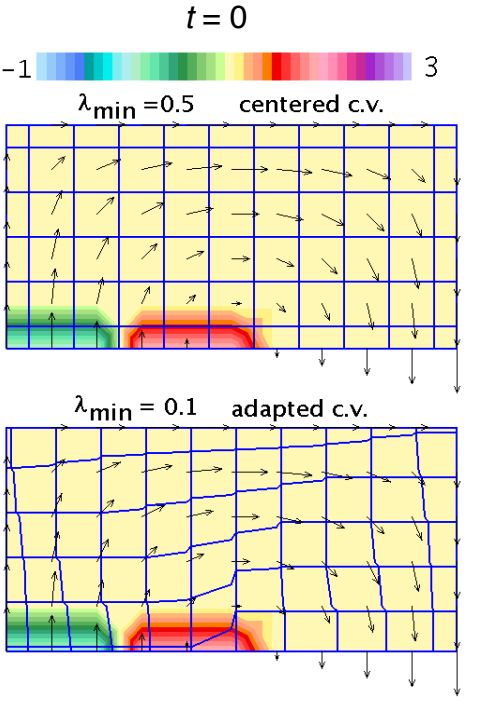
VIDEO: boxcav.5.1.mp4

Top - centered control volumes

Species concentration continues to wobble.

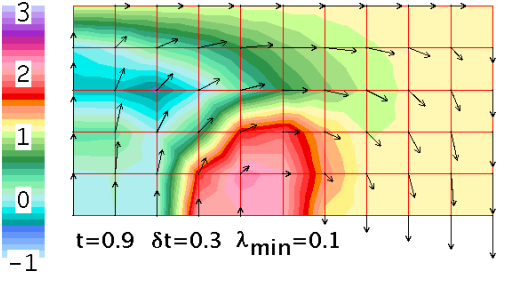
Bottom - convection adapted control volumes

Solution settles in the time it takes the flow to traverse from inlet to exit

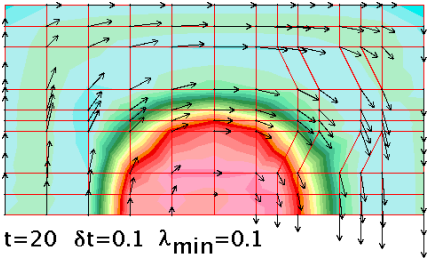


Variations

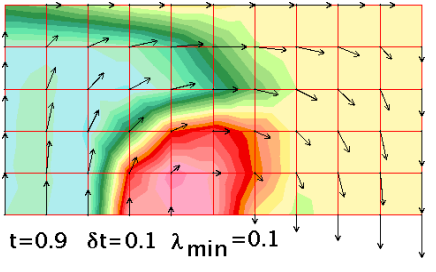
Increasing the timestep - times 3



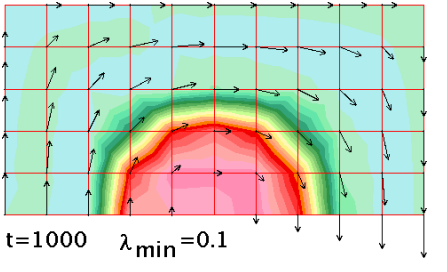
Messed-up grid



compared with

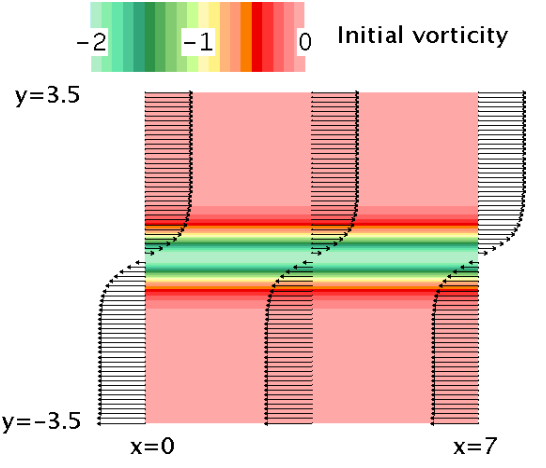


compared with uniform grid



Choose grid and timestep for physical resolution - not numerical limitations.

Kelvin-Helmholtz Instability



Solve continuity and momentum,

$$\frac{\partial u_i}{\partial x_i} = 0 \qquad \rho \frac{\partial u_k}{\partial t} + \rho u_i \frac{\partial u_k}{\partial x_i} = -\frac{\partial p}{\partial x_k}$$

using the predictor-corrector method iteratively for each time step.

Need explicit way of updating u_k for each grid point, each iteration, *m.*

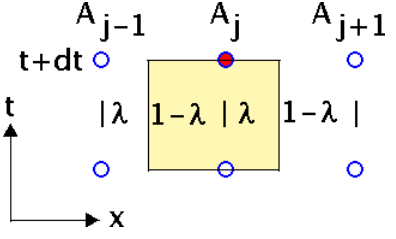
$$A_s(\hat{u}_k^{t+\delta t,m+1} - u_k^{t+\delta t,m}) = \int \left(-\rho \frac{\partial u_k}{\partial t} - \rho u_i \frac{\partial u_k}{\partial x_i} - \frac{\partial p}{\partial x_k}\right)^m dxdydz\, dt \quad => \hat{u}_k$$

$$A_s(u_k^{t+\delta t,m+1} - \hat{u}_k^{t+\delta t,m+1}) = -\int \frac{\partial (p^{m+1} - p^m)}{\partial x_k} dxdydzdt \quad \& \quad \frac{\partial u_k}{\partial x_k} = 0 \quad => u_k,\ p$$

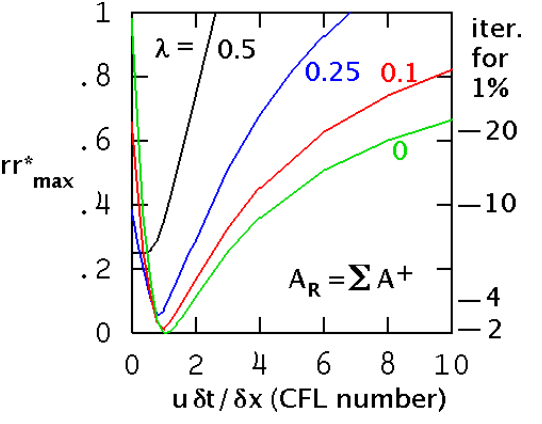
Center point coefficient A_s = ?

Center point coefficient, $A_s = A_R + A_c$

A_R to iterate for effects of non-center point coefficients at $t+\delta t$.



Stability analysis

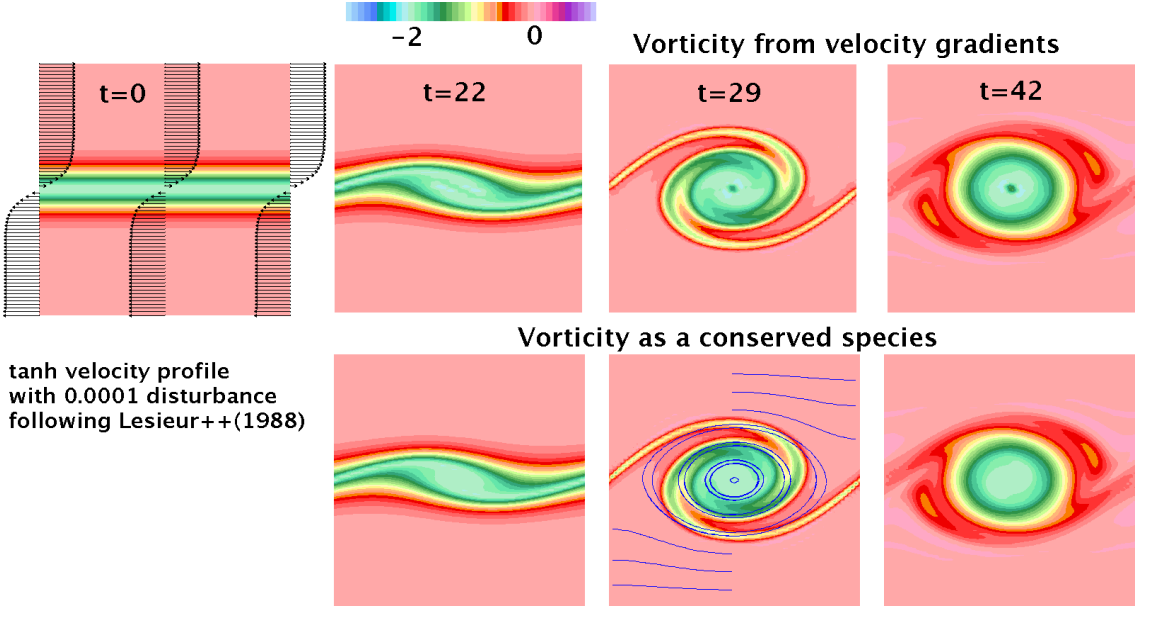


A_c to iterate for non-linearity of momentum.

Change in $\rho u_i \dfrac{\partial u_k}{\partial x_i}$: $\rho u_i \dfrac{\partial(\delta u_k)}{\partial x_i}$ is in coefs, A_j, but $\rho\,(\delta u_i)\dfrac{\partial u_k}{\partial x_i}$ is not.

$$A_c = \max_{i=1,2,3}\left(\int_{mom.c.v.} \rho \sum_{k=1}^{3}\sqrt{\left|\frac{\partial u_i}{\partial x_k}\right|\left|\frac{\partial u_k}{\partial x_i}\right|}\,dVol\right)dt$$

Kelvin-Helmholtz instability - development of vortex

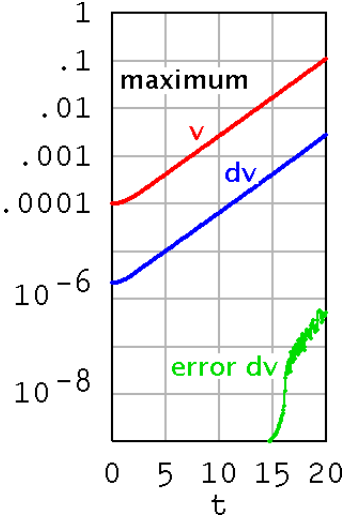


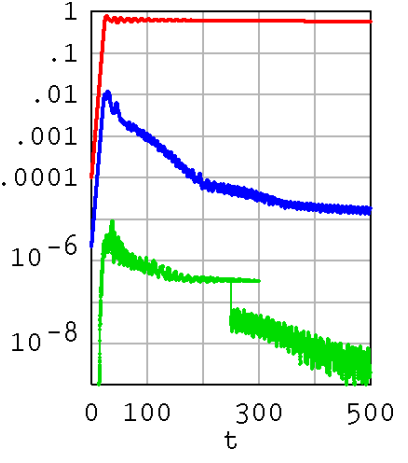
tanh velocity profile
with 0.0001 disturbance
following Lesieur++(1988)

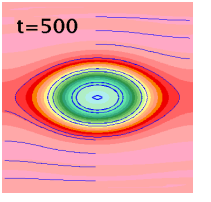
2-d inviscid unsteady flow:

vorticity from velocity gradients = vorticity from a conserved species equation

Kelvin -Helmholtz instability - growth rate







t=500,
Jupiter's spot ?!

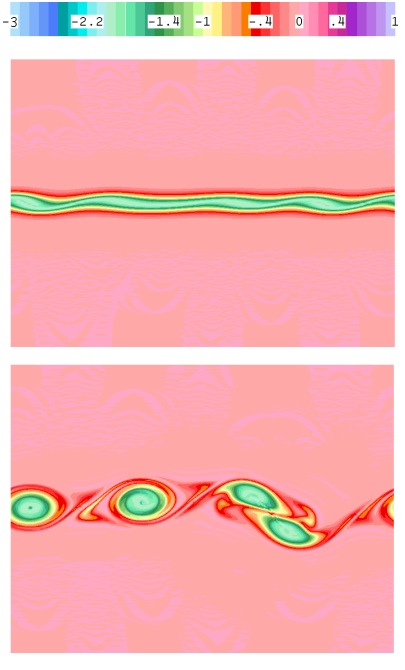
v-velocity has linear growth region: $\frac{d \ln v_{\max}}{dt} \Big/ \left(\frac{du}{dy}\right)_{\max, t=0}$ = 0.1887 Δx =7

compare with Michalke (1964) linear Eqs.: 0.1898 Δx =7.066

VIDEOS: kh28x21.mp4

28x21 domain, roundoff start

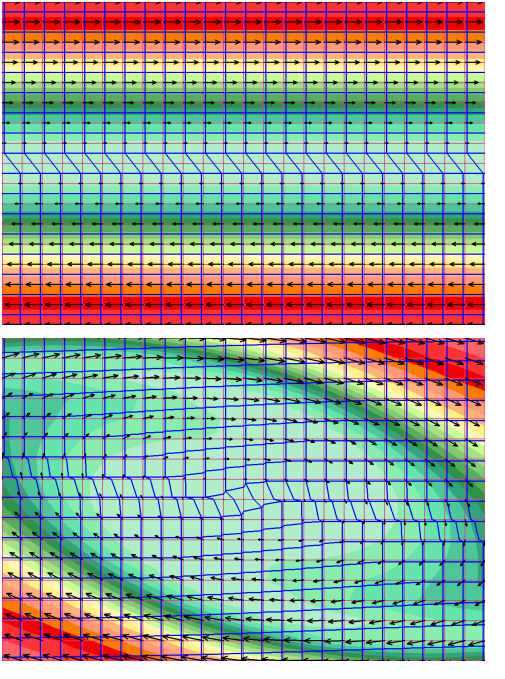
4 vortices develop and combine



kh.control.vol.mp4

Control volumes in vortex core

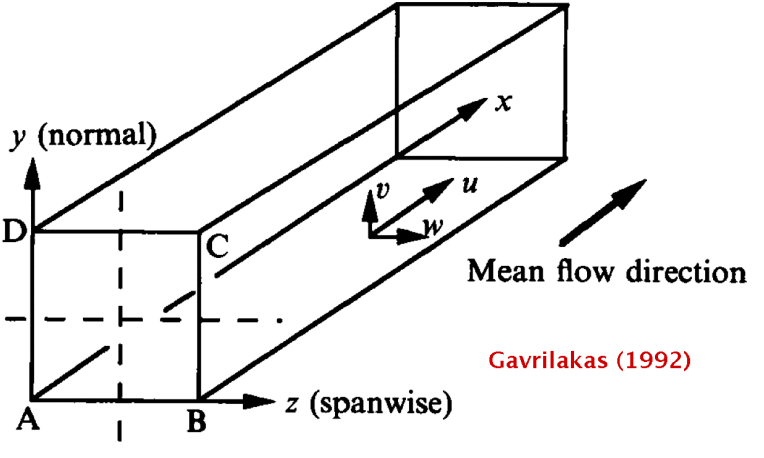
adapt as flow changes



Square Channel "DNS" $\mathrm{Re}^+ = \rho D \overline{u}_\tau / \mu = 300$

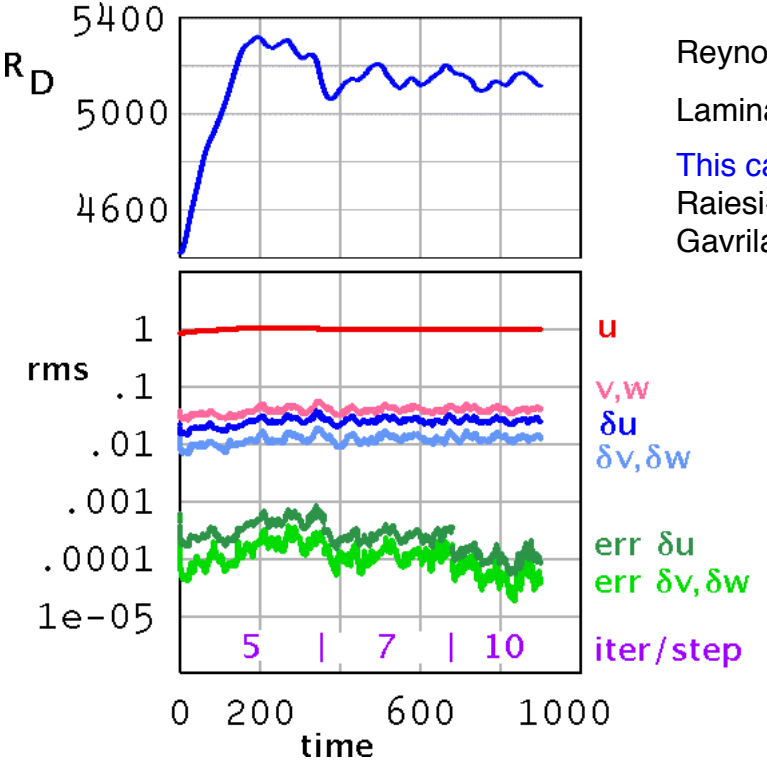
	$\Delta x, \Delta y, \Delta z$	Grid points	δx^+	$\delta y^+, \delta z^+$	δt
Gavrilakas (1992)	20π, 2,2	1000,127,127	9.4	0.46-4.6	CFL< 0.3
Raiesi++ (2011)	12, 2,2	361,181,181	5	< 2	CFL< 0.3
This paper (2014)	10.2, 2,2	52, 49, 49	30	1.5-7.5	~0.6 $\delta x / \overline{U}$

Will this coarse grid + adaptive c.v. approach give useful unsteady results which do not blow up and do not become laminar?



Gavrilakas (1992)

Convergence



Reynolds number = $\rho D \overline{U} / \mu$

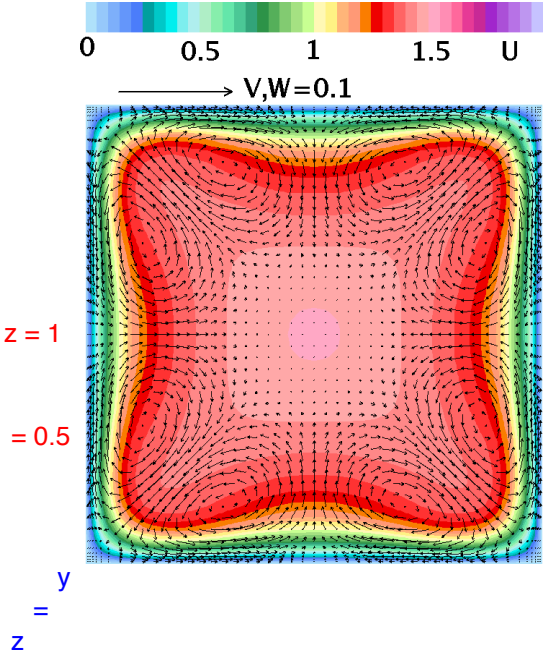
Laminar	6183
This calc.	5140
Raiesi++	4472
Gavrilakas	4410

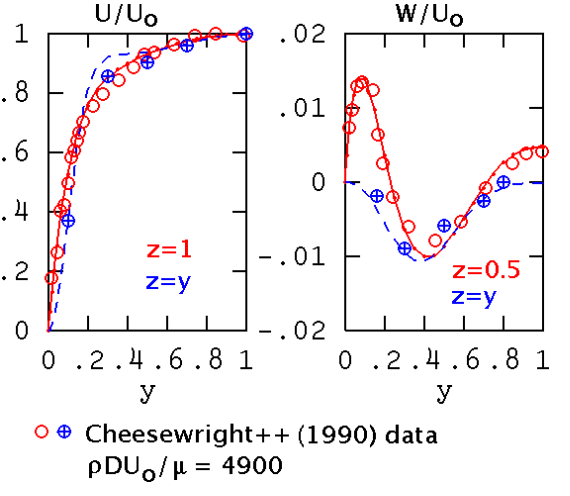
point-wise rms velocity

changes per timestep

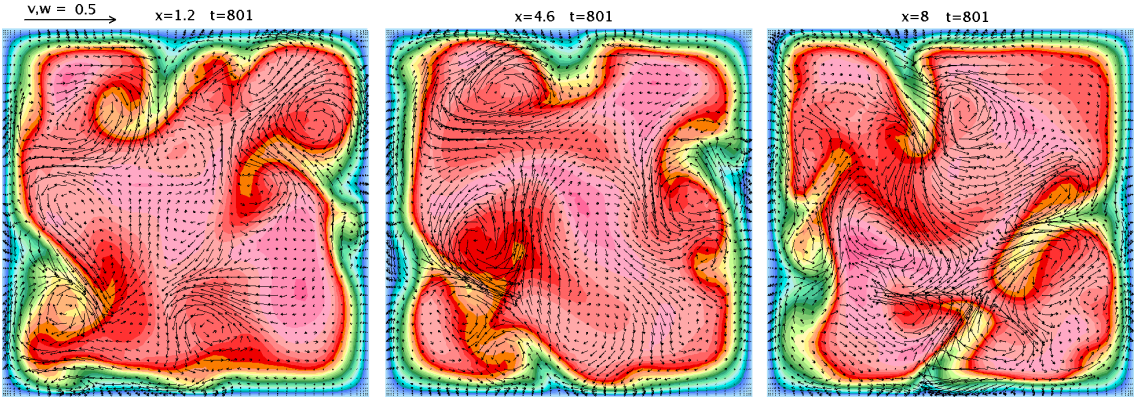
error in changes per timestep

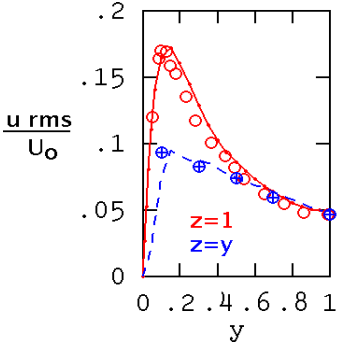
Mean Flow - Time / Ensemble Average Velocity Components

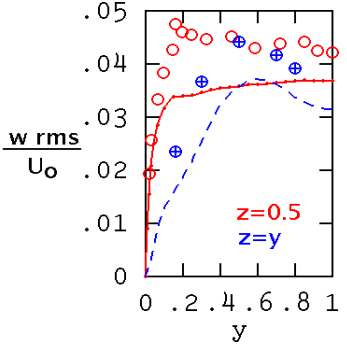




Unsteady Flow - Turbulence Statistics







Turbulence of primary flow well represented.

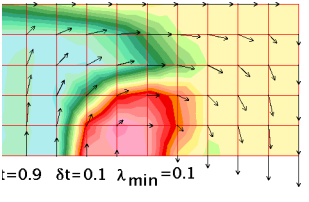
Turbulence of secondary flow qualitatively correct but low due to large axial grid spacing.

Summary - Method Includes

- Tri-linear profiles in space for convected and diffused properties
  => grid for physical resolution, NOT for orthogonality or spacing ratio
- Convection adapted control volumes
  => convection numerically stable *even without* time or diffusion terms
  => CFL condition becomes a time-accuracy parameter
- Full linear discretization of time term in space, and averages of convection and diffusion term over time step
  => 5 to 15 iterations used to converge each time step for CFL ~ 1

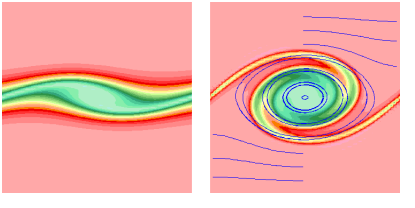
Test Cases

Box Cavity
Inviscid convection



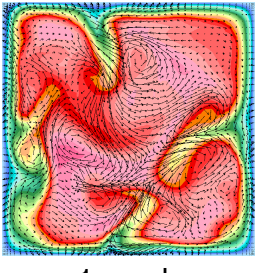
Mac mini CPU:
~ 1 second

Kelvin-Helmholtz Instability
Inviscid flow



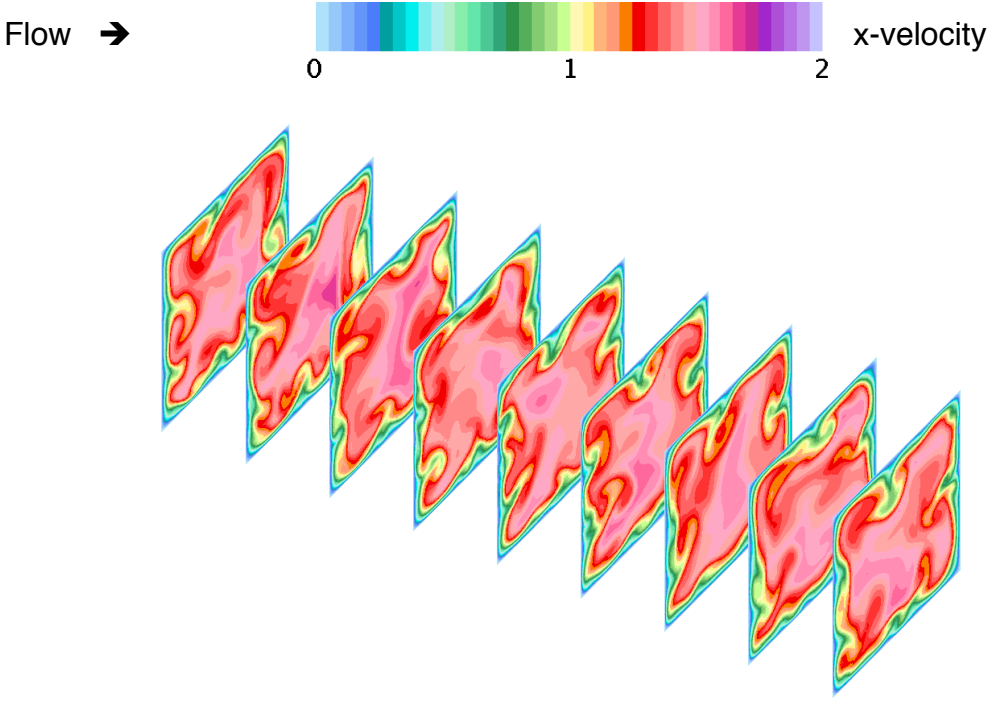
~ 45 minutes

Square Channel "DNS"
Laminar flow



~ 1 week

VIDEO: square.chan.mp4



Implications for LES

Current LES ???

1. Centered control volumes with 2nd order (or higher) discretization
   - *requires* Boussinesq subgrid scale model to stabilize the numerical instabilities.
2. Centered control volume with various upwind discretizations for stability.
   May introduce numerical mixing so that the subgrid scale model didn't matter.
3. Current results - good quantitative accuracy if grid within factor of 2 of DNS.

Way Forward - LES

Convection adapted control volumes -

No subgrid scale model needed for stability.

Real separation of scales **-** Linear profiles truncate subgrid scale variations.

Develop more realistic subgrid scale model - *not* Boussinesq

-> broader Reynolds number applicability ?

COMPRESSIBLE FLOW SHOCKTUBE EXAMPLE

Initial test in M4D, July 2015

Inviscid Fluid Flow Equations in Conservative Form

Continuity $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho U_j) = 0$$

Momentum $$\frac{\partial \rho U_i}{\partial t} + \frac{\partial}{\partial x_j}(\rho U_j U_i + \delta_{ij} p) = 0$$

Perfect gas $$p = \rho R T$$

Energy $$\frac{\partial \rho E}{\partial t} + \frac{\partial}{\partial x_j}\Big(U_j(\rho E + p)\Big) = 0$$ where $$E = c_v T + \frac{1}{2} U_k U_k$$

or $$\frac{\partial(\rho H - p)}{\partial t} + \frac{\partial}{\partial x_j}\Big(\rho U_j H\Big) = 0$$ where $$H = c_p T + \frac{1}{2} U_k U_k$$

Test Case setup from Hirsch (Vol. 2, 1990, Fig 16.6.8 refers to Sod, 1978)

Initial conditions for 1-d space, time calculation.

$p = 10^5, \quad \rho = 1, \quad U = 0$ for $x < x_o$;

$p = 10^4, \quad \rho = 0.125, \quad U = 0$ for $x > x_o$.

Discretize using linear profiles with adaptive control volumes, $\lambda_{\min} = 0.1$

Exception: ρRT interpolated to mid-way between the points is used in the momentum equation for p on the upstream side of the control volume. (First order approximation for stability. A higher order interpolated ρRT would be better but is not coded.)

Results

___ theory

…. Calc.

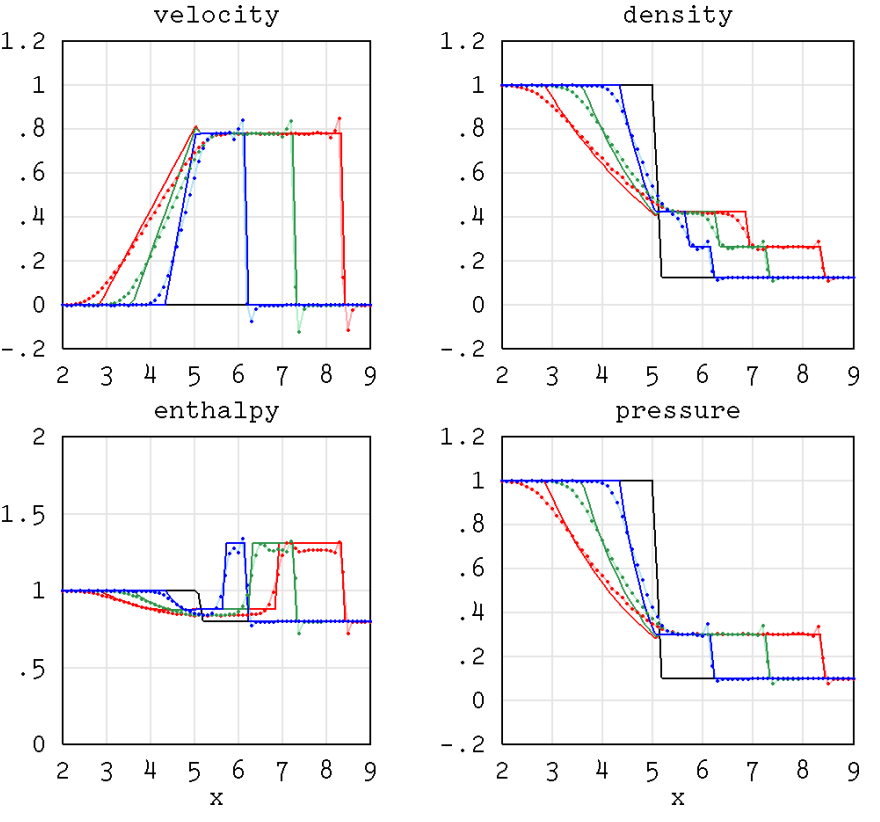
for t=0, 0.002, 0.004, 0.006

normalized initial $x<x_o$ and $U/c_{x<xo}$

dx = 0.1

dt = 0.00001

(0.085 CFL)



M4D - input commands -> procedure:

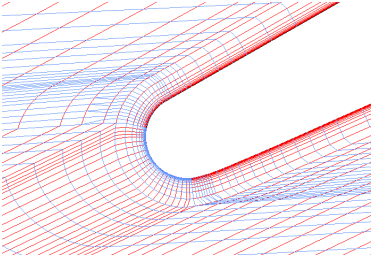
Time accurate or steady

Inviscid, laminar or turbulent

Incompressible -

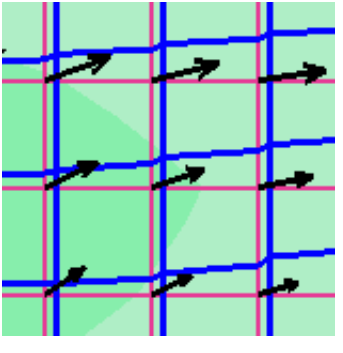
pressure correction method

Single block grid

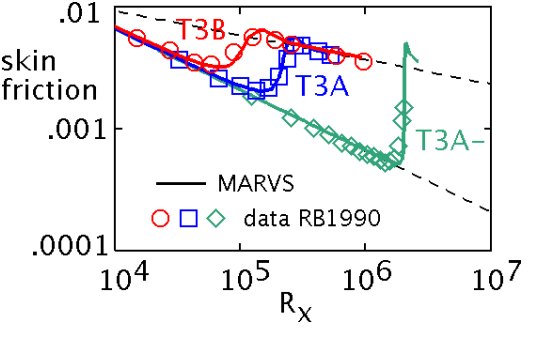


Paradigm shift features:

3-d linear profiles over convection adapted control volumes



MARVS Reynolds stress model transition - shear flow at high S/ω

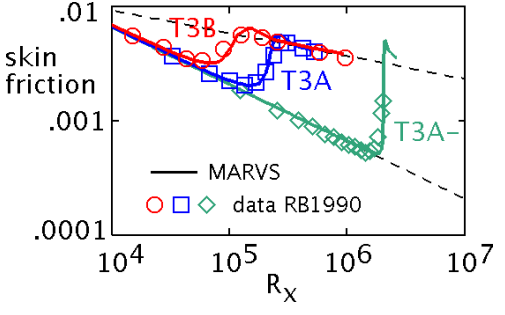


Part 2
BOUNDARY LAYER TRANSITION
WITH THE MARVS REYNOLDS STRESS MODEL

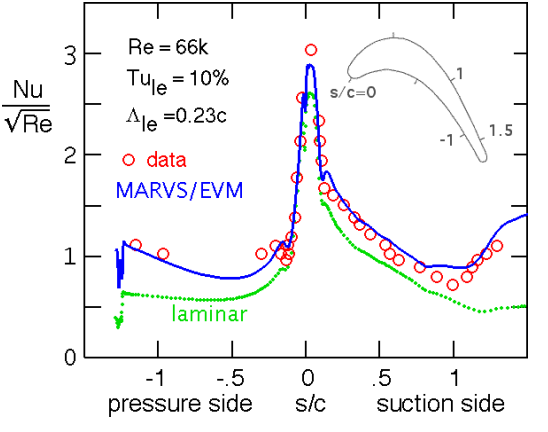
Joan G. Moore and John Moore

Paper ETC 2015-113

Flat Plate - thin leading edge



Turbine Cascade Heat Transfer



Model development did NOT include transitional boundary layers.

Why does the model work?

Incompressible RANS continuity & momentum

$$\frac{\partial U_i}{\partial x_i} = 0 \qquad U_k \frac{\partial U_i}{\partial x_k} - \frac{\partial}{\partial x_k} \nu \frac{\partial U_i}{\partial x_k} = -\frac{\partial \overline{u_i u_k}}{\partial x_k} - \frac{1}{\rho}\frac{\partial p}{\partial x_i}$$

Closed with MARVS, 7-eq turbulence model for $\overline{u_i u_k}$

$$U_i \frac{\partial \varphi}{\partial x_i} - \frac{\partial}{\partial x_i} \nu \frac{\partial \varphi}{\partial x_i} - \frac{\partial}{\partial x_i} \nu_{t_{ij}} \frac{\partial \varphi}{\partial x_j} = S_\varphi ,$$

$\varphi =$

turb. velocity scale

$$q = \sqrt{k}$$

dissipation frequency

$$\omega = \frac{\tilde{\varepsilon}}{k} = \frac{\left(\varepsilon - 2\nu\left(\frac{\partial q}{\partial x_i}\right)^2\right)}{k}$$

turb. anisotropy (5)

$$b_{ij} = \frac{\overline{u_i u_j}}{2k} - \frac{\delta_{ij}}{3}$$

$$= \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix}$$

Viewing the Reynolds stress tensor as velocity variations due to turbulence

2-d flow Cartesian coordinates

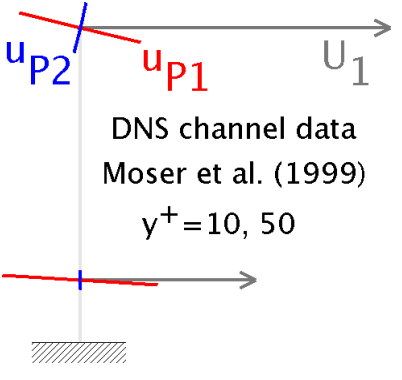
$$\overline{u_i u_j} = \begin{bmatrix} \overline{u_1 u_1} & \overline{u_1 u_2} & 0 \\ \overline{u_1 u_2} & \overline{u_2 u_2} & 0 \\ 0 & 0 & \overline{u_3 u_3} \end{bmatrix}$$

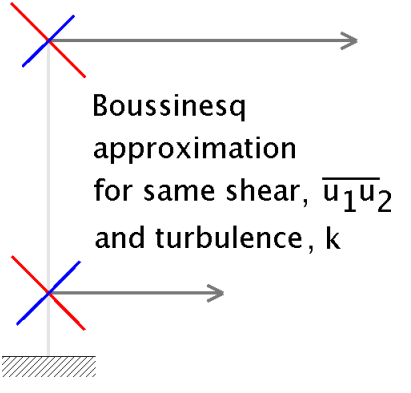
In Principal coordinates

$$\overline{u_{Pi} u_{Pj}} = \begin{bmatrix} \overline{u_{P1} u_{P1}} & 0 & 0 \\ 0 & \overline{u_{P2} u_{P2}} & 0 \\ 0 & 0 & \overline{u_{P3} u_{P3}} \end{bmatrix}$$

Assume $u_{P1} = \hat{u}_{P1} \sin B_1 t$ gives $\hat{u}_{P1} = \sqrt{2\overline{u_{P1} u_{P1}}}$

and u_{P1} varies between $\pm \hat{u}_{P1}$ in the $P1$ direction.



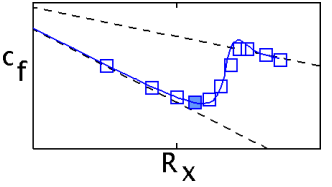


ERCOFTAC
case T3A

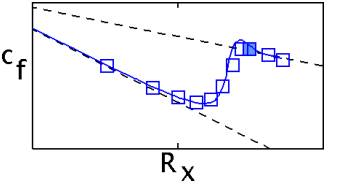
at x=0:

Tu_k = 2.9%

$R_t = \frac{k}{\omega\nu}$ =127

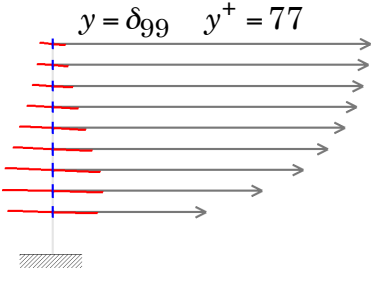


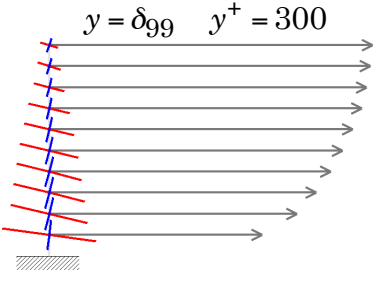
start x=0.395m



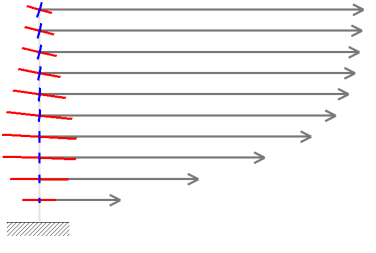
end x=0.895m

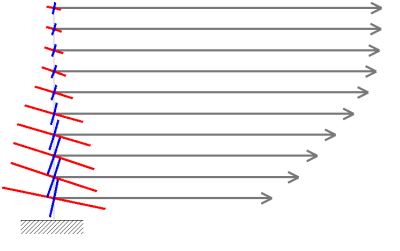
Data
RB1990





Calc.
MARVS



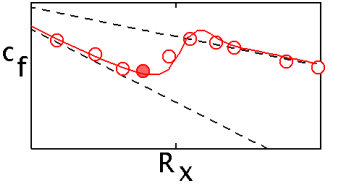


ERCOFTAC
case T3B

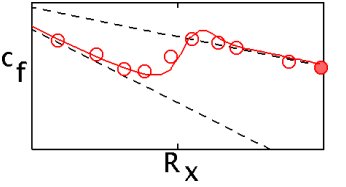
at x=0:

Tu_k = 5.3%

$R_t = \frac{k}{\omega\nu}$ =1133



start x=0.095m

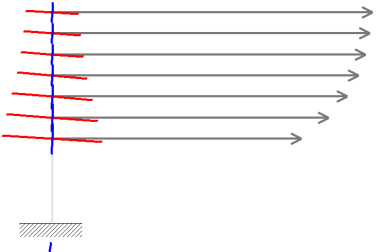


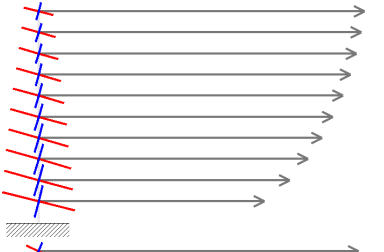
fully turb. x=1.495m

$y = \delta_{99}$ $y^+ = 64$

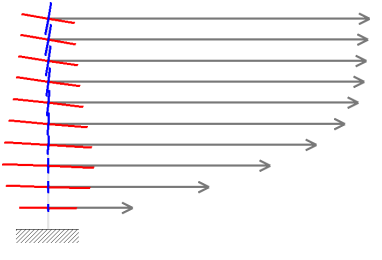
$y = \delta_{99}$ $y^+ = 850$

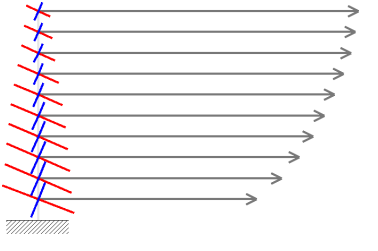
Data
RB1990





Calc.
MARVS



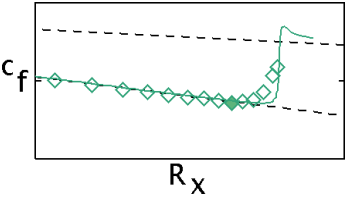


ERCOFTAC case T3A–

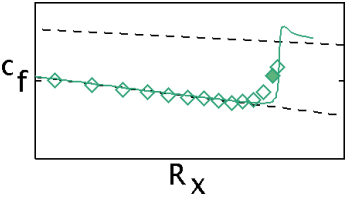
at x=0:

Tu_k = 0.8%

$R_t = \frac{k}{\omega\nu}$ =78



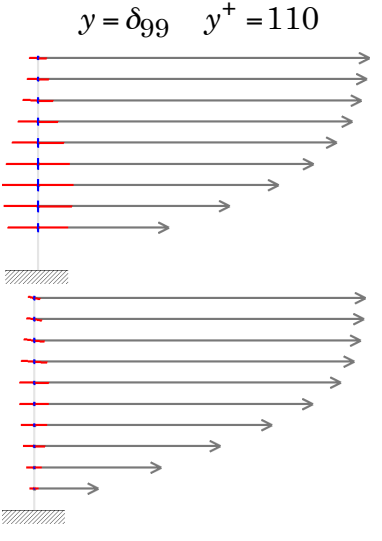
start x=1.095m

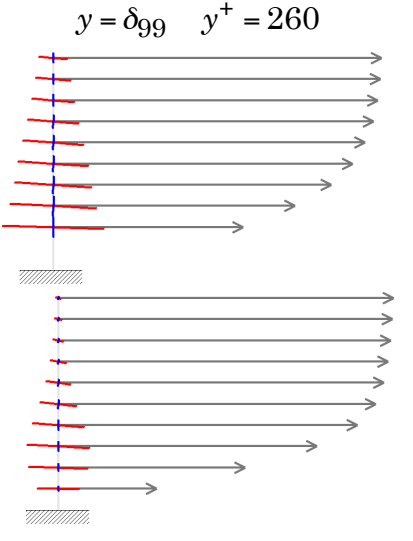


mid x=1.495m

Data RB1990

Calc. MARVS





Pre-transition and near-wall turbulence is streaky. Why?

Lee, Kim and Moin (1990), homogeneous shear flow DNS (***no walls !!***) observed that their

results for high dimensionless strain rate, S/ω, are like near-wall turbulence.

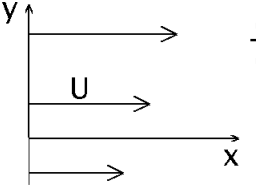
And like rapid distortion theory.

Homogeneous Shear Flow

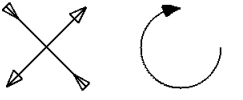
$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

$$W_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right)$$

$$S = \sqrt{2 S_{ij} S_{ij}}$$

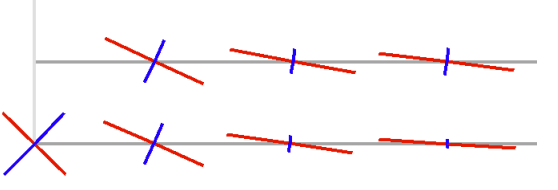


$\frac{dU}{dy}$ = strain & vorticity



$\frac{S}{\omega} \sim 17$ LKM DNS

$\frac{S}{\omega} = \infty$ RDT



dimensionless time St = 0.05 2 6 12

Vorticity ***counter***-rotates the Reynolds stress tensor more when S/ω is high.

Effect of Principal Stress Orientation on Turbulence Production

Turbulence production rate: $P = -\overline{u_i u_j} S_{ij}$ $\quad S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$

Reynolds stress model

$$\overline{u_i u_j} = 2k b_{ij} + 2k\delta_{ij}/3$$

$$P = -2k b_{ij} S_{ij}$$

2-Equation model

$$\overline{u_i u_j} = -2\nu_t S_{ij} + 2k\delta_{ij}/3$$

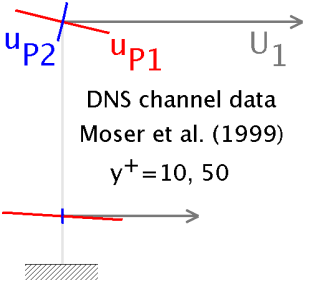
$$P = 2\nu_t S_{ij} S_{ij}$$

near wall and pre-transition have low *P*

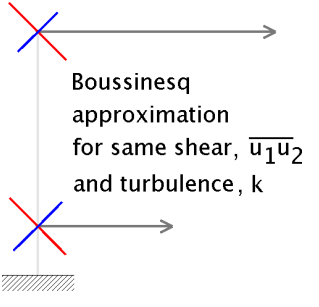
$b_{ij}S_{ij}$ correlates poorly

$S_{ij}S_{ij}$ correlates perfectly, extra modeling to lower ν_t

$\frac{S}{\omega} = 3.7$



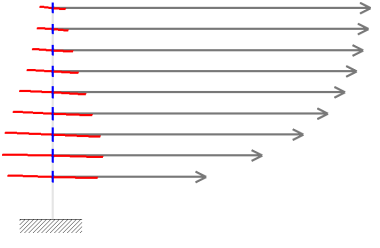
$\frac{S}{\omega} = 18$



ERCOFTAC FLAT PLATE TEST CASES

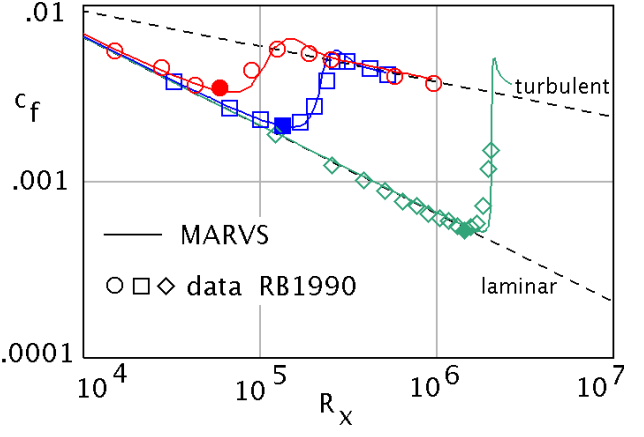
Initial Conditions x=0

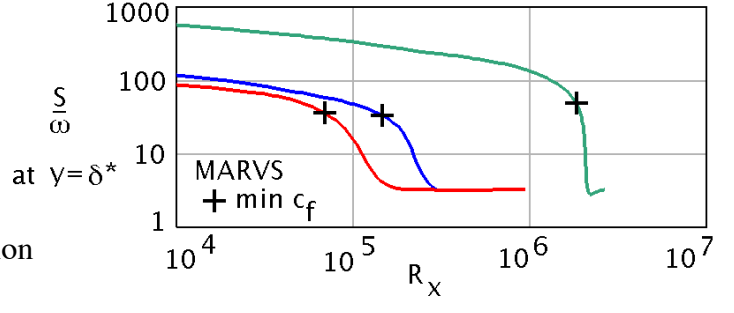
Case	T3B	T3A	T3A–
Tu_k%	5.28	2.94	0.78
$R_t = \frac{k}{\omega\nu}$	1133	127	78
b_{11}	0.072	0.066	0.080
b_{22}	-0.129	-0.069	-0.088



Laminar: $\delta^* \sim \delta_{99}/3$

Turbulent: δ^* in log-law region





MARVS Reynolds stress model, source term for b_{ij} equations

$$S_{bij} = -\frac{1}{2}b_{ij}(c_1-2)\omega - \frac{1}{2}b_{ij}(2+\frac{1}{2}c_6)\frac{P}{k} + \frac{1}{2}\left(c_3-\frac{4}{3}\right)S_{ij}$$

$$+\frac{1}{2}(c_4-2)(b_{ik}S_{jk}+b_{jk}S_{ik}-\frac{2}{3}\delta_{ij}b_{mn}S_{mn}) + \frac{1}{2}(c_5-2)(b_{ik}W_{jk}+b_{jk}W_{ik})$$

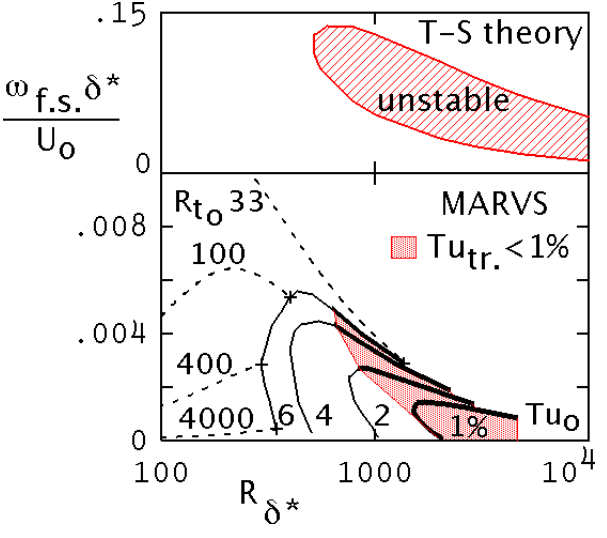
c_1 models slow pressure-strain and anisotropy of dissipation.
Note term is relatively small when S/ω is large.

c_3, c_4, c_5, c_6 model rapid pressure-strain. $c_5 = 0$. $c_6 = 0$ for shear flow.

M - Moore
A - anisotropy of Reynolds stresses
R - rapidly distorting or fully developed Reynolds stresses
V - vorticity – shear flow or irrotational flow
S - splat wall boundary condition for b_{ij}

Data base: Experiments, DNS, **rapid distortion theory** and theoretical limits.
Homogeneous turbulence, with/without vorticity and coordinate rotation.
Fully developed pipe and channel flows.

Relationship ? to linear stability theory and Tollmien-Schlichting waves



Unstable frequencies
for a Blasius profile
from Schlichting, 1975

MARVS flat plate transition
Vary Tu_o and R_{t_o}

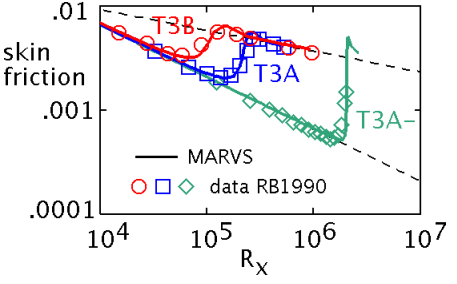
$\omega_{\text{f.s.}}$ ω (model) in the free stream.

- - - - - pre-transition trajectories for $Tu_o = 6\%$.

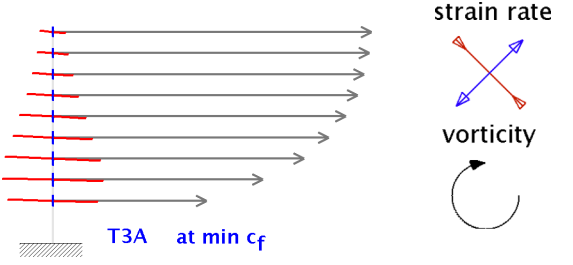
——— start of transition

Linear stability theory considers velocity profiles which are instantaneously inflectional due to the fluctuating velocities. As well as in boundary layers, these also occur in uniform shear flows and are a (hidden) part of homogeneous turbulence modeling, i.e. the MARVS model.

Flat Plate - thin leading edge.



Why does the model work?



Turbulence near walls and in pre-transitional boundary layers is streaky.

This is primarily due to the effect high dimensionless strain rate, S/ω, has on homogeneous turbulence - *not the proximity of a wall.*

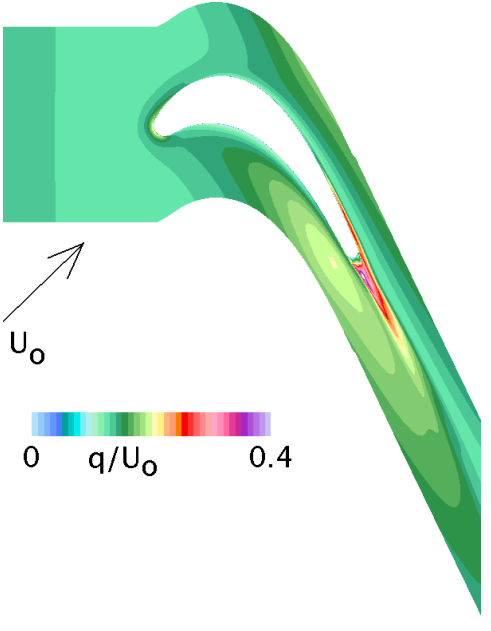
The **vorticity** in the shear flow, turns the Reynolds stress tensor in the opposite direction more when S/ω is high.

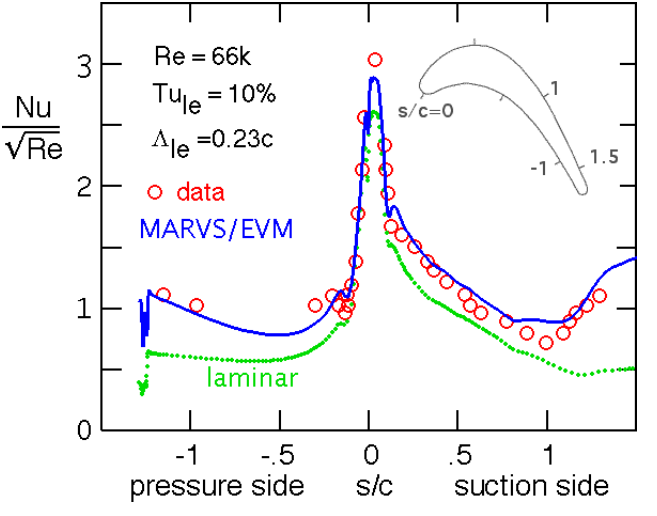
Turbulence production is reduced due to the poor correlation between the principal directions of the tensors $-\overline{u_i u_j}$ and S_{ij}.

The MARVS model captures this essential homogeneous turbulence physics.

2-d Turbine Cascade Heat Transfer

MARVS model -> flow, q, ω. Use with EVM model, Pr=0.7, uniform heat flux

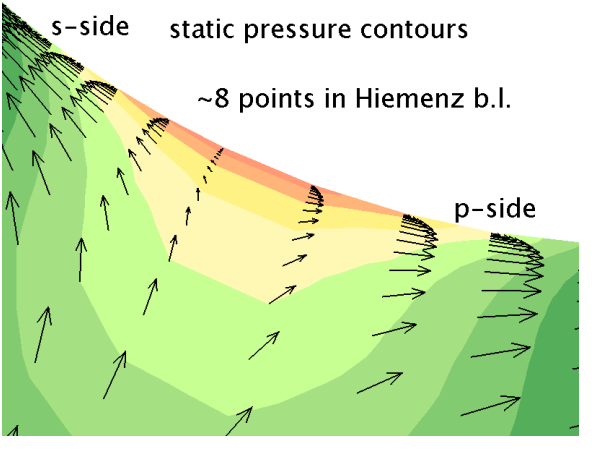


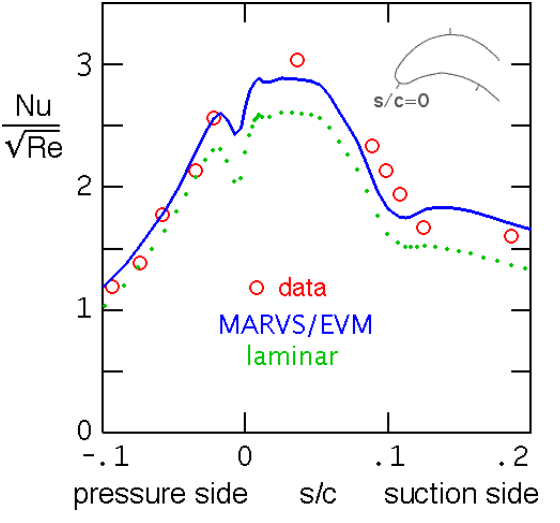


o Butler et al. (2001) data

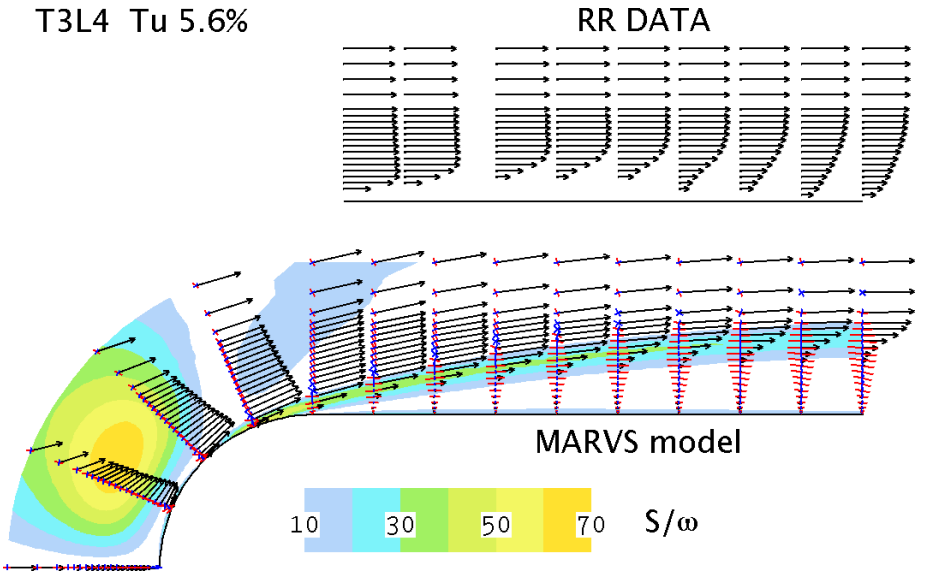
laminar using MARVS model velocities

…. Turbine Cascade Leading Edge





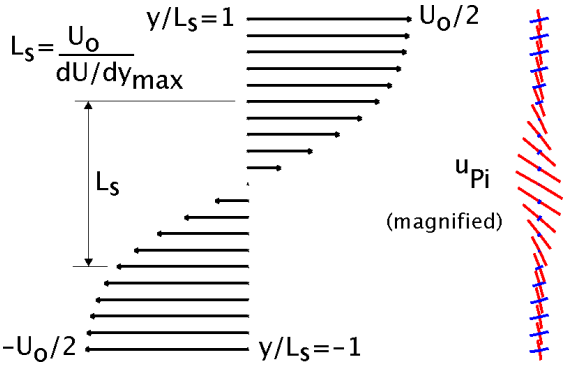
Way Forward - Reynolds Stress Model - Separated Flow

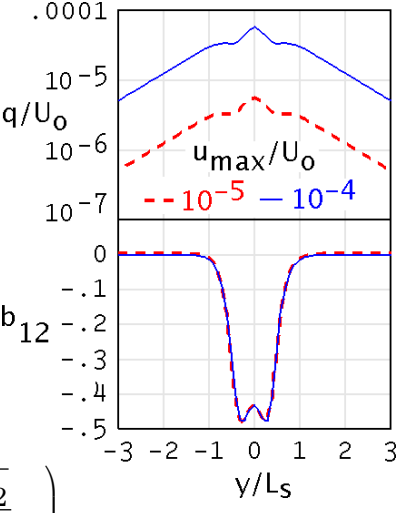


Classic inflectional velocity profile is unstable.

This **inhomogeneous** effect is NOT included in MARVS development.

Analyze inviscid Kelvin-Helmholtz instability for turbulence (x-average)





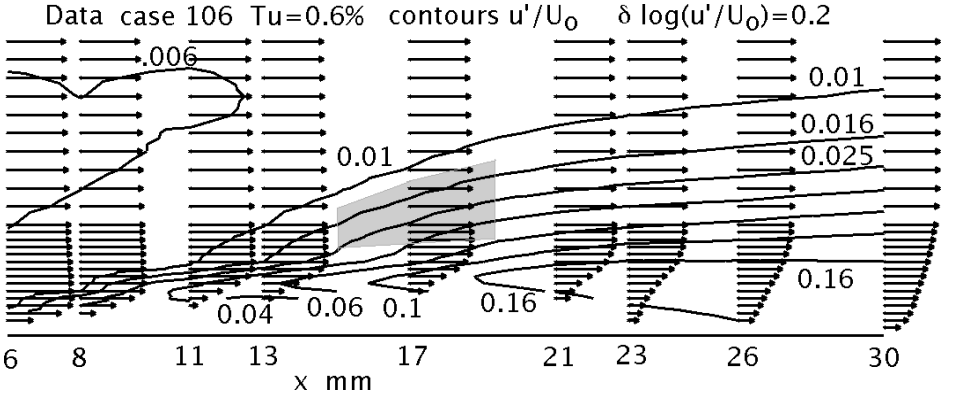
Exact k-equation balance

$$\frac{1}{k\partial U/\partial y_{\max}}\left(\frac{\partial k}{\partial t} = -2kb_{12}\frac{\partial U}{\partial y} - \frac{1}{2}\frac{\partial \overline{u_i u_i u_2}}{\partial y} - \frac{1}{\rho}\frac{\partial \overline{pu_2}}{\partial y} \right)$$

$\frac{y}{L_s} = 0$	0.38 =	0.86	– small	– 0.48
$\left\lvert\frac{y}{L_s}\right\rvert > 1.5$	0.38 =	0	– small	+ 0.38

Large pressure diffusion. Outside shear layer ln*q* linear in *y*.

Does data show Kelvin-Helmholtz instability turbulence? **YES!**



Kelvin-Helmholtz instability ?

(a) Does log u' increase linearly in *x* in center of shear layer ? Yes

	Inviscid 2-d	T3L 0.6% data
$(D\ln u'/Dt)/(dU/dy)_{\max}$ =	0.19	0.064

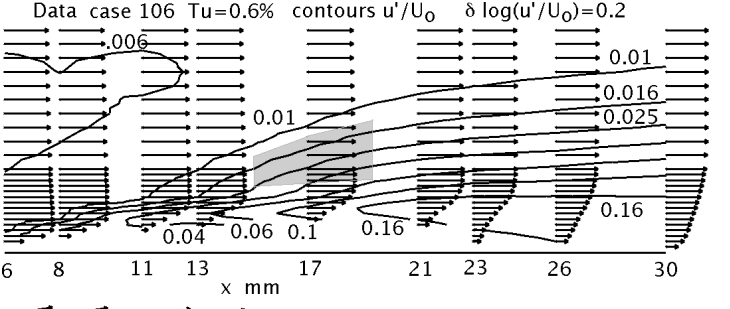
(b) Does log u' vary linearly in *y* outside the shear layer ? Yes

$\nu_t = c_{p.d.} U_o^2/(dU/dy)_{\max}$ $c_{p.d.}$ =	0.23	0.08

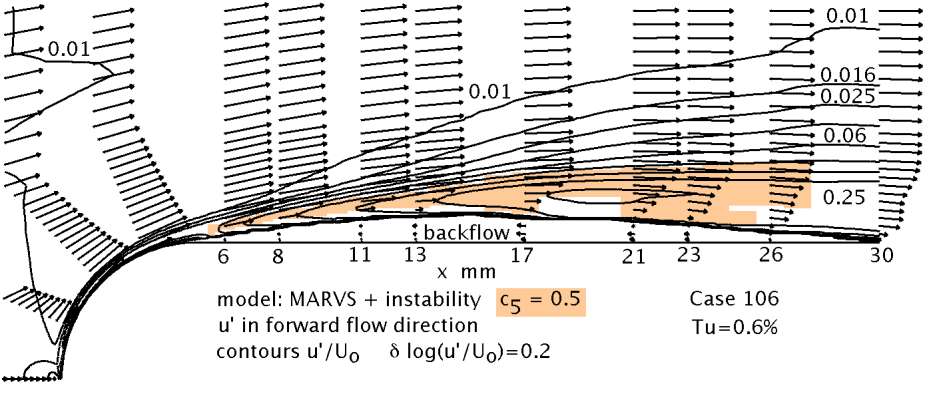
Preliminary Instability Model: Use search procedure to find instability regions.
Cancel out some turning by vorticity with c_5 pressure-strain.
Add pressure-diffusion of k and ω, **but not** b_{ij}, over larger region.

T3L case Tu=0.6%

DATA



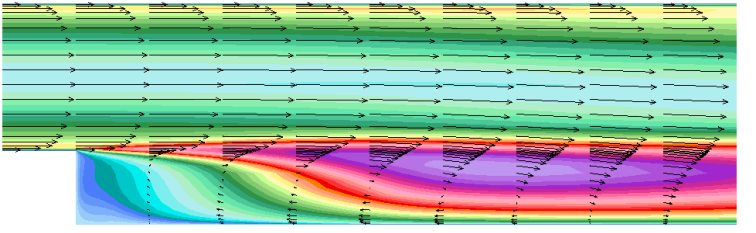
MARVS + Instability model



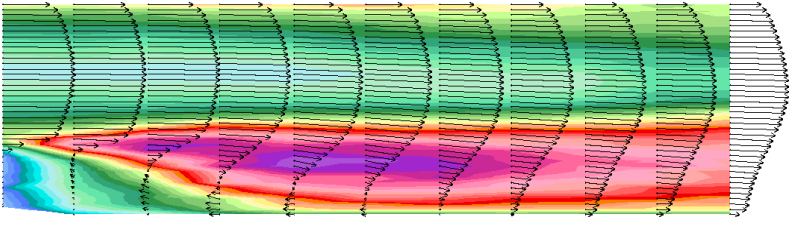
Kasagi 2:1 Backstep - Rd = 9550, fully developed turbulent inlet

Compare q/U_{max}

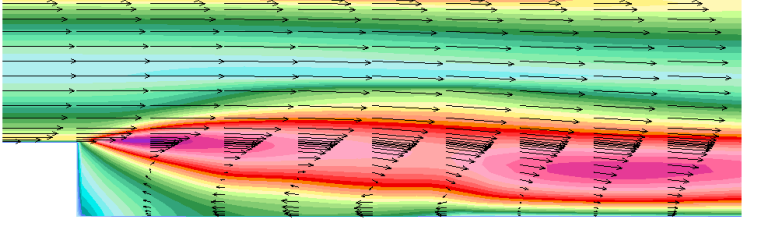
MARVS



Kasagi Data (1993)



MARVS + Instability model



M4D
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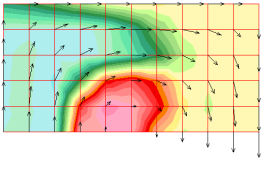
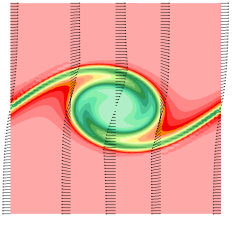
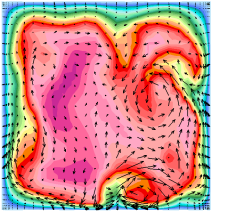
CLASSICAL AND TURBULENT FLOWS

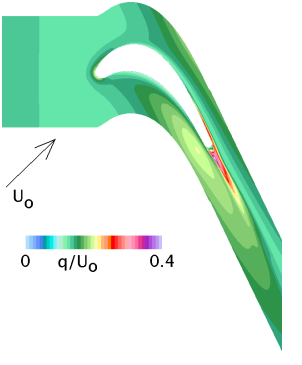
moore64.home.comcast.net/toolbox/m4d.2015.2.tar.gz

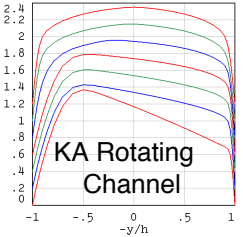
Open source C code. Imbedded plot package. Documentation.
Input and sample output for 6 ready-to-run examples.





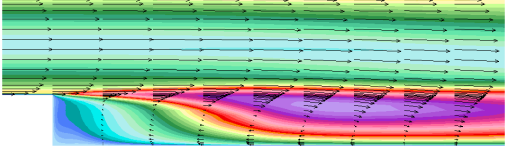






KA Rotating Channel

Um for KA Ro=0 .01 .05 .10 .20 .50
each slipped by 0.2



Resources

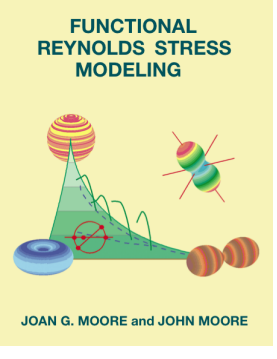
Website: **moore64.home.comcast.net**

Click on Papers and Presentations - recent papers etc.

Software

M4D: moore64.home.comcast.net/toolbox/m4d.2015.2.tar.gz

Flat Plate Transition: moore64.home.comcast.net/toolbox/flatplate.jgm.tar



Book: **Functional Reynolds Stress Modeling**

Joan G. Moore and John Moore, Oct. 2006

Available from:

Amazon.com

In several libraries

M4D - Documentation

What you can do:

Compile the C code (see m4d.whatshere.setup.run.pdf) then

Try the given examples - see doc.rtf for each example

Modify the examples or make up you own examples -
see m4d.commands.pdf and m4d.commands.short.pdf

For the programmer:
Add commands to M4D to extend its capabilities -
see m4d.subroutines.pdf and m4d.variables.pdf

M4D Input - Command Format (ignored comments recolored)

```
c: comment ******** box cavity: convection around 180 degrees (half vortex)

c: constant alim d 1 .1        control volume limit parameter
c: constant dt d 1 .1          time step

c: infile inn.grid11x6 1       set up grid
c: infile inn.init 1           initialize variables
c: infile inn.init.coefLL 1    set up control volumes and coefs

-------- setups and initial plots ----------
c: constant p.defaultdir s 1 ../../jgm          default dir for color.map etc
c: arraydump out/converge alim dt ""            start convergence file
c: infile inn.plotcv 1                          plot control volumes   out/cv0.gif

c: constant ITERPLOT i 1 1        time steps between cc plots

-------------------  start calculation -----------------
c: infile inn.step.o2c  50          take 50  2nd-order time steps
c: arraydump out/cc50 cc ""        save concentration on file
```

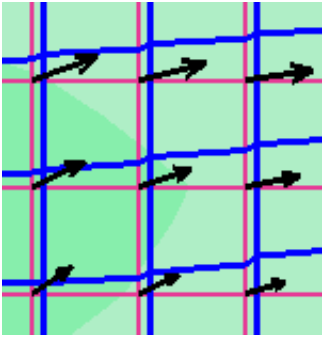
M4D Differential Equations

Continuity $\partial \rho U_i / \partial x_i = 0$ steady or incompressible.

Momentum - inviscid, laminar or turbulent, steady or unsteady.
Select needed terms.

$$\rho \frac{\partial U_i}{\partial t} + \rho U_k \frac{\partial U_i}{\partial x_k} - \frac{\partial}{\partial x_k} \mu \frac{\partial U_i}{\partial x_k} - \frac{\partial}{\partial x_k} \mu_t \frac{\partial U_i}{\partial x_k} =$$

$$-\frac{\partial \rho \overline{u_i u_k}}{\partial x_k} - \frac{\partial p}{\partial x_i} - \delta_{i1} \frac{\partial p}{\partial x} - 2\rho \varepsilon_{i3m} \Omega_3 U_m + \rho \frac{\partial}{\partial x_i} \left(\frac{\Omega^2 r^2}{2} \right) + \frac{\partial}{\partial x_k} (\mu + \mu_t) \frac{\partial U_k}{\partial x_i}$$



ρ, U_i at grid points

Continuity control volumes, ρU_i 3d linear

Momentum control volumes, U_i 3d linear

p at corners of momentum control volumes
one point in each continuity control volume

Pressure-correction solution procedure

M4D Differential Equations cont.

Turbulence model properties, generic form. Select needed terms.

$$\rho\frac{\partial\phi}{\partial t}+\rho U_i\frac{\partial\phi}{\partial x_i}-\frac{\partial}{\partial x_i}\mu\frac{\partial\phi}{\partial x_i}-\frac{\partial}{\partial x_i}\mu_t\frac{\partial\phi}{\partial x_i}-\frac{\partial}{\partial x_i}\mu_{t_{ik}}\frac{\partial\phi}{\partial x_k}=S_\phi$$

Models:

Reynolds stress (MARV or MARVS) $\phi = q, \omega, b_{ij}$

2 Eq. (Coakley with modified ω bndry. condition) $\phi = q, \omega$

Conserved species, including energy. Select needed terms.

$$\rho\frac{\partial\phi}{\partial t}+\rho U_i\frac{\partial\phi}{\partial x_i}-\frac{\partial}{\partial x_i}\frac{\mu}{\mathrm{Pr}}\frac{\partial\phi}{\partial x_i}-\frac{\partial}{\partial x_i}\frac{\mu_t}{\mathrm{Pr}_t}\frac{\partial\phi}{\partial x_i}=0$$

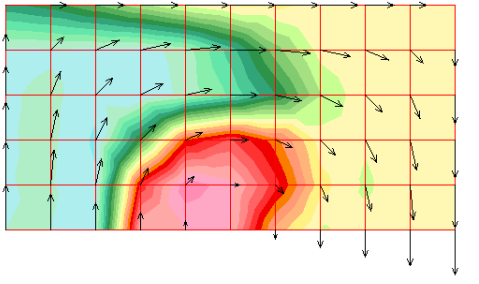
Model: MARV/MARVS-compatible EVM for μ_t/Pr_t using q, ω, Pr

Use convection adapted control volumes, like for momentum.

M4D Imbedded Plot Package

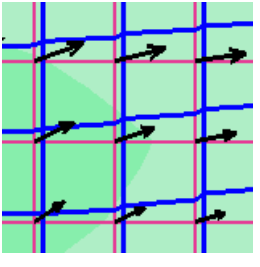
Color fill contours,
velocity vectors, and/or grid lines.

Plot each timestep to create frames for videos. (.gif files)

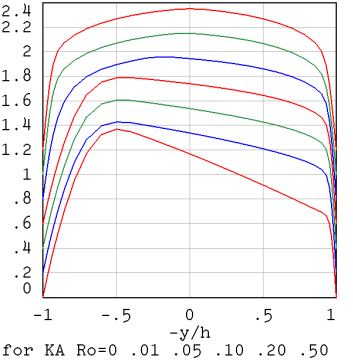


Labeled color bars.

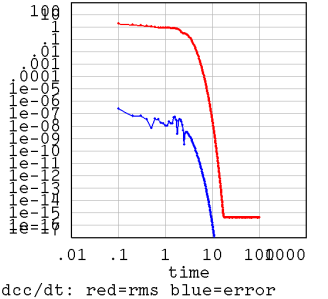
Control volumes.



Line plots - linear or log scales



U/Um for KA Ro=0 .01 .05 .10 .20 .50
each slipped by 0.2



dcc/dt: red=rms blue=error

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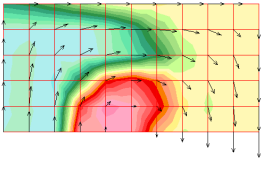

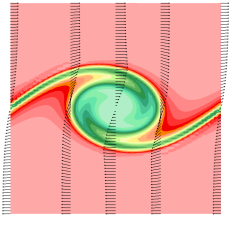

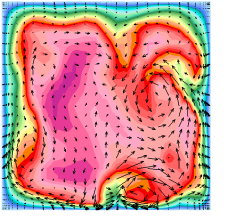

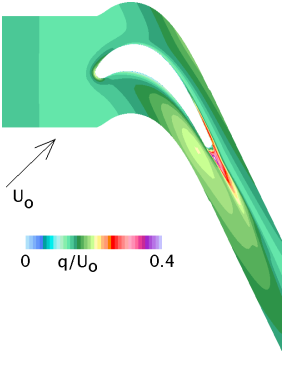

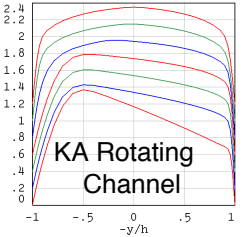

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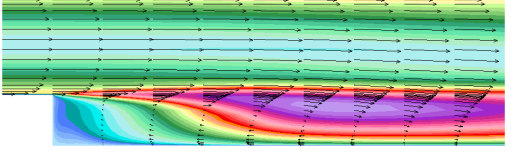
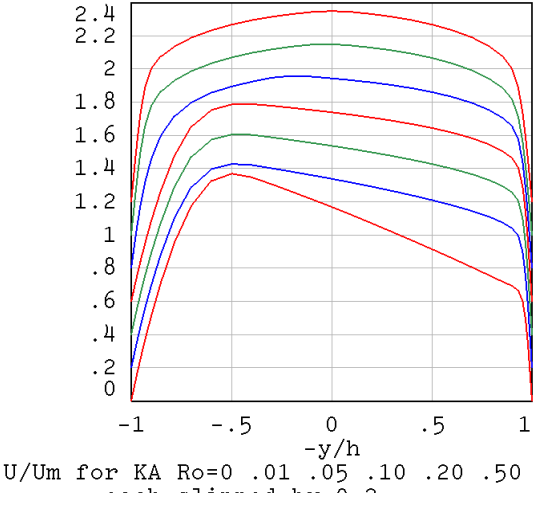
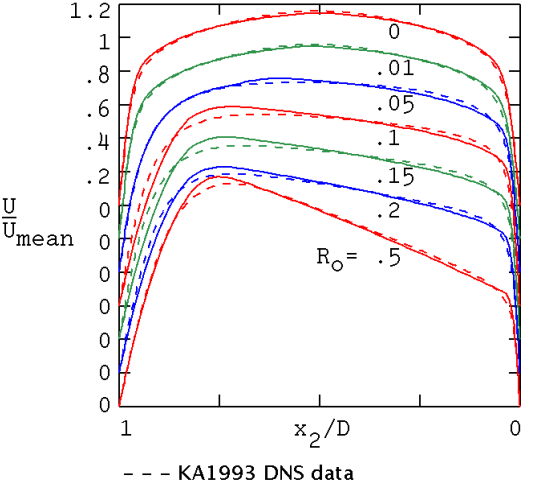


MARVS Reynolds stress model tests:

2-D Fully Developed Rotating Channel - steady turbulent flow, U_x momentum and Reynolds stress equations.

Compare U/U_m M4D MARVS with MARV and KA1993 DNS





M4D - input commands -> procedure:

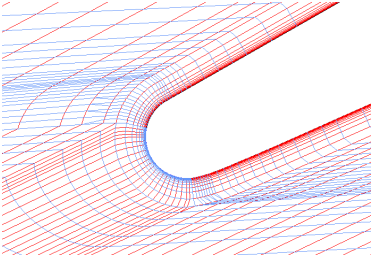
Time accurate or steady

Inviscid, laminar or turbulent

Incompressible -

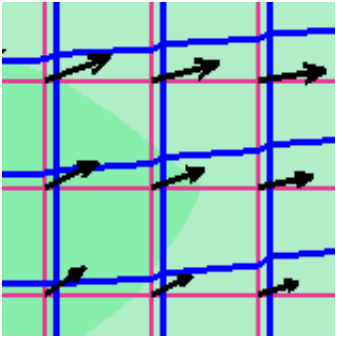
pressure correction method

Single block grid



Paradigm shift features:

3-d linear profiles over convection adapted control volumes



MARVS Reynolds stress model transition - shear flow at high S/ω