M4D
AN OPEN SOURCE RESEARCH CFD CODE
FOR THE CALCULATION OF
CLASSICAL AND TURBULENT FLOWS

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Presentation at NIA
August 2015
M4D - **input commands -> procedure:**

Time accurate or steady
Inviscid, laminar or turbulent
Incompressible - pressure correction method

**Paradigm shift features:**
3-d linear profiles over convection adapted control volumes

**Single block grid**

**MARVS Reynolds stress model transition - shear flow at high** $S/\omega$
Part 1:
USING MULTI-DIMENSIONAL LINEAR DISCRETIZATION OVER UNSTEADY CONVECTION ADAPTED CONTROL VOLUMES

Joan G. Moore and John Moore

AIAA Paper 2014-2780

Box Cavity                 Kelvin-Helmholtz Instability                 Square Channel “DNS”

\( t=0.9 \), \( \delta t=0.1 \), \( \lambda_{\text{min}}=0.1 \)
Space marching
Upstream volume

\[ T \text{ 3-d linear} \]
Stable - 2nd order

\[ -2 \quad 2 \quad 0 \]

\[ -12 \quad 12 \quad 0 \]

\[ A_{\text{center}} / \sum A^+ = 0.75 \]

Take fully 3-D?
Centered volume

\[ T \text{ 3-d linear} \]
Unstable

\[ -1 \quad 0 \quad 1 \]

\[ -6 \quad 6 \]

\[ A_{\text{center}} / \sum A^+ = 0 \]

Fully 3-D solution
Centered volume

\[ T \text{ stepwise, upwind} \]
Stable - 1st order

\[ -16 \quad 16 \quad 0 \]

\[ A_{\text{center}} / \sum A^+ = 1 \]
MEFP - 1984 - 3d steady code

\[ U_i \frac{\partial T}{\partial x_i} = 0 \]

3-D linear T profiles

1/8th volumes are assigned to give control volumes which are upstream of the points.

Stable - 2nd order

MEFP control volumes

IAHR Box Cavity Convection Test
Given velocity and T-inlet

MEFP results
MEFP grid for turbine cascade
not orthogonal
spacing ratio mostly < 2
ok with 3-d linear profiles
This paper - M4D - 2014 - unsteady (4d) code
Continuous convection adapted control volumes for unsteady flow.

**In space:**
Control volume divide location $\lambda_i$ determined for each between-the-points volume for each time step based on average velocity, $u_i$, and viscosity.

Move divide location from center towards sides with incoming flow.

Limit $\lambda_i$: $\lambda_{\text{min}} \leq \lambda_i \leq 1 - \lambda_{\text{min}}$, where $0 < \lambda_{\text{min}} \leq 0.5$

$\Rightarrow$ control volume surfaces do not coincide with grid surfaces.
Box cavity test - unsteady transient

Solve \[ \frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = 0 \]

Use: \[ \delta t = \frac{\delta x}{u_{\max}} = 0.1 \]

Convergence
Transient solutions show dispersion/mixing errors. Best choice: $\lambda_{\text{min}} = 0.1$

timesteps: 2  9  10,000

$\lambda_{\text{min}}$

0.49

0.25

0.1

0.01
VIDEO: boxcav.5.1.mp4

Top - centered control volumes
Species concentration continues to wobble.

Bottom - convection adapted control volumes
Solution settles in the time it takes the flow to traverse from inlet to exit
Variations

Increasing the timestep - times 3

Messed-up grid

compared with

compared with uniform grid

Choose grid and timestep for physical resolution - not numerical limitations.
Kelvin-Helmholtz Instability

Solve continuity and momentum,

\[
\frac{\partial u_i}{\partial x_i} = 0 \quad \rho \frac{\partial u_k}{\partial t} + \rho u_i \frac{\partial u_k}{\partial x_i} = - \frac{\partial p}{\partial x_k}
\]

using the predictor-corrector method iteratively for each time step.

Need explicit way of updating \( u_k \) for each grid point, each iteration, \( m \).

\[
A_s(\hat{u}_k^{t+\delta t,m+1} - u_k^{t+\delta t,m}) = \int \left( -\rho \frac{\partial u_k}{\partial t} - \rho u_i \frac{\partial u_k}{\partial x_i} - \frac{\partial p}{\partial x_k} \right)^m \, dx \, dy \, dz \, dt \quad \Rightarrow \quad \hat{u}_k^{t+\delta t,m+1}
\]

\[
A_s(u_k^{t+\delta t,m+1} - \hat{u}_k^{t+\delta t,m+1}) = -\int \frac{\partial (p^{m+1} - p^m)}{\partial x_k} \, dx \, dy \, dz \, dt \quad \& \quad \frac{\partial u_k}{\partial x_k} = 0 \quad \Rightarrow \quad u_k, \ p
\]

Center point coefficient \( A_s = ? \)
Center point coefficient, \( A_s = A_R + A_c \)

\( A_R \) to iterate for effects of non-center point coefficients at \( t + \delta t \).

\[
\begin{array}{ccc}
A_{j-1} & A_j & A_{j+1} \\
t + \delta t & & \\
& |\lambda| & 1-\lambda & |\lambda| & 1-\lambda & \\
\end{array}
\]

\( A_c \) to iterate for non-linearity of momentum.

Change in \( \rho u_i \frac{\partial u_k}{\partial x_i} \): \( \rho u_i \frac{\partial (u_k)}{\partial x_i} \) is in coefs, \( A_j \), but \( \rho \frac{\partial u_i}{\partial x_i} \frac{\partial u_k}{\partial x_i} \) is not.

\[
A_c = \max_{i=1,2,3} \left( \int_{\text{mom.c.v.}} \rho \sum_{k=1}^{3} \sqrt{\frac{\partial u_i}{\partial x_k} \frac{\partial u_k}{\partial x_i}} \ dVol \right) dt
\]
Kelvin-Helmholtz instability - development of vortex

Vorticity from velocity gradients

Vorticity as a conserved species

tanh velocity profile with 0.0001 disturbance following Lesieur++ (1988)

2-d inviscid unsteady flow:
\[ \text{vorticity from velocity gradients} = \text{vorticity from a conserved species equation} \]
v-velocity has linear growth region:  
\[
\frac{d \ln v_{\text{max}}}{dt} \left/ \left( \frac{du}{dy} \right)_{\text{max}, t=0} \right. = 0.1887 \quad \Delta x = 7
\]

compare with Michalke (1964) linear Eqs.: 0.1898  \( \Delta x = 7.066 \)
VIDEOS: kh28x21.mp4
28x21 domain, roundoff start
4 vortices develop and combine

kh.control.vol.mp4
Control volumes in vortex core
adapt as flow changes
Square Channel “DNS” \[ \text{Re}^+ = \frac{\rho D \bar{u}_r}{\mu} = 300 \]

\[ \Delta x, \Delta y, \Delta z \quad \text{Grid points} \quad \delta x^+, \delta y^+, \delta z^+ \quad \delta t \]

<table>
<thead>
<tr>
<th>Author</th>
<th>( x, y, z ) Grid points</th>
<th>( \delta x^+ )</th>
<th>( \delta y^+, \delta z^+ )</th>
<th>( \delta t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gavrilakas (1992)</td>
<td>20, 2, 2</td>
<td>1000, 127, 127</td>
<td>9.4</td>
<td>0.46-4.6</td>
</tr>
<tr>
<td>Raiesi++ (2011)</td>
<td>12, 2, 2</td>
<td>361, 181, 181</td>
<td>5</td>
<td>&lt; 2</td>
</tr>
<tr>
<td>This paper (2014)</td>
<td>10.2, 2, 2</td>
<td>52, 49, 49</td>
<td>30</td>
<td>1.5-7.5</td>
</tr>
</tbody>
</table>

Will this coarse grid + adaptive c.v. approach give useful unsteady results which do not blow up and do not become laminar?
Reynolds number = $\rho D \overline{U} / \mu$

Laminar: 6183
This calc.: 5140
Raiesi++: 4472
Gavrilakas: 4410

Point-wise rms velocity changes per timestep
Error in changes per timestep
Mean Flow - Time / Ensemble Average Velocity Components

$z = 1$

$z = 0.5$

$y = z$

$V, W = 0.1$

$U/U_0$ for $z=1$

$W/U_0$ for $z=0.5$

Cheesewright++ (1990) data

$\rho DU_0/\mu = 4900$
Turbulence of primary flow well represented.

Turbulence of secondary flow qualitatively correct but low due to large axial grid spacing.
Summary - Method Includes

- **Tri-linear profiles in space** for convected and diffused properties
  
  => grid for physical resolution, **NOT** for orthogonality or spacing ratio

- **Convection adapted control volumes**
  
  => convection numerically stable *even without* time or diffusion terms
  
  => CFL condition becomes a time-accuracy parameter

- **Full linear discretization of time term in space**, and averages of convection and diffusion term over time step
  
  => 5 to 15 iterations used to converge each time step for CFL ~ 1

Test Cases

<table>
<thead>
<tr>
<th>Box Cavity</th>
<th>Kelvin-Helmholtz Instability</th>
<th>Square Channel “DNS”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inviscid convection</td>
<td>Inviscid flow</td>
<td>Laminar flow</td>
</tr>
</tbody>
</table>

Mac mini CPU:

- ~ 1 second
- ~ 45 minutes
- ~ 1 week
VIDEO: square.chan.mp4

Flow →

x-velocity
Implications for LES

Current LES ???

1. Centered control volumes with 2nd order (or higher) discretization
   - requires Boussinesq subgrid scale model to stabilize the numerical instabilities.

2. Centered control volume with various upwind discretizations for stability.
   May introduce numerical mixing so that the subgrid scale model didn’t matter.

3. Current results - good quantitative accuracy if grid within factor of 2 of DNS.

Way Forward - LES

Convection adapted control volumes -
   No subgrid scale model needed for stability.
Real separation of scales - Linear profiles truncate subgrid scale variations.
Develop more realistic subgrid scale model - not Boussinesq
   -> broader Reynolds number applicability ?
COMPRESSIBLE FLOW SHOCKTUBE EXAMPLE

Initial test in M4D, July 2015

Inviscid Fluid Flow Equations in Conservative Form

Continuity
\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho U_j) = 0 \]

Momentum
\[ \frac{\partial \rho U_i}{\partial t} + \frac{\partial}{\partial x_j} (\rho U_j U_i + \delta_{ij} p) = 0 \]

Perfect gas
\[ p = \rho RT \]

Energy
\[ \frac{\partial \rho E}{\partial t} + \frac{\partial}{\partial x_j} \left( U_j (\rho E + p) \right) = 0 \]
where
\[ E = c_v T + \frac{1}{2} U_k U_k \]

or
\[ \frac{\partial (\rho H - p)}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho U_j H \right) = 0 \]
where
\[ H = c_p T + \frac{1}{2} U_k U_k \]
Test Case setup from Hirsch (Vol. 2, 1990, Fig 16.6.8 refers to Sod, 1978)

Initial conditions for 1-d space, time calculation.

\[ p = 10^5, \quad \rho = 1, \quad U = 0 \quad \text{for} \quad x < x_0; \]
\[ p = 10^4, \quad \rho = 0.125, \quad U = 0 \quad \text{for} \quad x > x_0. \]

Discretize using linear profiles with adaptive control volumes, \( \lambda_{\text{min}} = 0.1 \)

Exception: \( \rho RT \) interpolated to mid-way between the points is used in the momentum equation for \( p \) on the upstream side of the control volume. (First order approximation for stability. A higher order interpolated \( \rho RT \) would be better but is not coded.)
Results

theory
.... Calc.
for $t=0, 0.002, 0.004, 0.006$

normalized initial $x < x_0$ and $U/c_{x < x_0}$

dx = 0.1
dt = 0.00001
(0.085 CFL)
M4D - **input commands -> procedure:**
- Time accurate or steady
- Inviscid, laminar or turbulent
- Incompressible - pressure correction method

**Paradigm shift features:**
- 3-d linear profiles over convection adapted control volumes
- MARVS Reynolds stress model transition - shear flow at high $S/\omega$
Part 2
BOUNDARY LAYER TRANSITION
WITH THE MARVS REYNOLDS STRESS MODEL

Joan G. Moore and John Moore
Paper ETC 2015-113

Flat Plate - thin leading edge

Model development did NOT include transitional boundary layers.

Why does the model work?
Incompressible RANS continuity & momentum

\[ \frac{\partial U_i}{\partial x_i} = 0 \]

\[ U_k \frac{\partial U_i}{\partial x_k} - \nu \frac{\partial^2 U_i}{\partial x_k \partial x_k} = - \frac{\partial u_i u_k}{\partial x_k} - \frac{1}{\rho} \frac{\partial p}{\partial x_i} \]

Closed with MARVS, 7-eq turbulence model for \( u_i u_k \)

\[ U_i \frac{\partial \varphi}{\partial x_i} - \nu \frac{\partial \varphi}{\partial x_i} - \nu_t \frac{\partial \varphi}{\partial x_j} = S_\varphi, \]

\[ \varphi = \]

\[ q = \sqrt{k} \]

\[ \omega = \tilde{\varepsilon} = \left( \varepsilon - 2\nu \left( \frac{\partial q}{\partial x_i} \right)^2 \right) \]

\[ b_{ij} = \frac{u_i u_j}{2k} - \frac{\delta_{ij}}{3} \]

\[ = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix} \]
Viewing the Reynolds stress tensor as velocity variations due to turbulence

2-d flow Cartesian coordinates

\[ u_iu_j = \begin{bmatrix} u_1u_1 & u_1u_2 & 0 \\ u_1u_2 & u_2u_2 & 0 \\ 0 & 0 & u_3u_3 \end{bmatrix} \]

In Principal coordinates

\[ u_{p_i}u_{p_j} = \begin{bmatrix} u_{p1}u_{p1} & 0 & 0 \\ 0 & u_{p2}u_{p2} & 0 \\ 0 & 0 & u_{p3}u_{p3} \end{bmatrix} \]

Assume \( u_{p1} = \hat{u}_{p1} \sin B_1 t \) gives \( \hat{u}_{p1} = \sqrt{2u_{p1}u_{p1}} \)

and \( u_{p1} \) varies between \( \pm \hat{u}_{p1} \) in the \( P1 \) direction.

DNS channel data
Moser et al. (1999)
\( y^+=10, 50 \)

Boussinesq approximation for same shear, \( \overline{u_1u_2} \) and turbulence, \( k \)
ERCOFTAC case T3A
at x=0:

\[ Tu_k = 2.9\% \]

\[ R_t = \frac{k}{\omega v} = 127 \]

start \( x=0.395\)m                     end \( x=0.895\)m

\[ y = \delta_{99} \quad y^+ = 77 \]

\[ y = \delta_{99} \quad y^+ = 300 \]

Data
RB1990

Calc.
MARVS
ERCOFTAC case T3B

at x=0:

\[ Tu_k = 5.3\% \]

\[ Re = \frac{k}{\omega v} = 1133 \]

start x=0.095m

fully turb. x=1.495m

\[ y = \delta_{99} \quad y^+ = 64 \]

\[ y = \delta_{99} \quad y^+ = 850 \]

Data RB1990

Calc. MARVS
ERCOFTAC case T3A– at x=0:

\[ Tu_k = 0.8\% \]
\[ R_l = \frac{k}{\nu} = 78 \]

Start: \( x = 1.095\,\text{m} \)

Mid: \( x = 1.495\,\text{m} \)

\[ y = \delta_{99} \quad y^+ = 110 \]

\[ y = \delta_{99} \quad y^+ = 260 \]

Data: RB1990

Calc. MARVS
Pre-transition and near-wall turbulence is streaky. Why?

Lee, Kim and Moin (1990), homogeneous shear flow DNS (no walls !!) observed that their results for high dimensionless strain rate, $S/\omega$, are like near-wall turbulence. And like rapid distortion theory.

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)
\]

\[
W_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right)
\]

\[
S = \sqrt{2S_{ij}S_{ij}}
\]

\[
\frac{S}{\omega} \sim 17
\]

\[
\frac{S}{\omega} = \infty
\]

dimensionless time $St = 0.05$ 2 6 12

Vorticity counter-rotates the Reynolds stress tensor more when $S/\omega$ is high.
Effect of Principal Stress Orientation on Turbulence Production

Turbulence production rate:

\[ P = -u_i u_j S_{ij} \]

\[ S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \]

Reynolds stress model

\[ u_i u_j = 2kb_{ij} + 2k\delta_{ij} / 3 \]

\[ P = -2kb_{ij}S_{ij} \]

2-Equation model

\[ u_i u_j = -2\nu_t S_{ij} + 2k\delta_{ij} / 3 \]

\[ P = 2\nu_t S_{ij}S_{ij} \]

near wall and pre-transition have low \( P \)

\( b_{ij}S_{ij} \) correlates poorly

\[ \frac{S}{\omega} = 3.7 \]

DNS channel data

Moser et al. (1999)

\( y^+ = 10, 50 \)

\[ \frac{S}{\omega} = 18 \]

Boussinesq approximation for same shear, \( u_1u_2 \) and turbulence, \( k \)
ERCOFTAC FLAT PLATE TEST CASES

Initial Conditions $x=0$

<table>
<thead>
<tr>
<th>Case</th>
<th>T3B</th>
<th>T3A</th>
<th>T3A–</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Tu_k%$</td>
<td>5.28</td>
<td>2.94</td>
<td>0.78</td>
</tr>
<tr>
<td>$R_t = \frac{k}{\omega v}$</td>
<td>1133</td>
<td>127</td>
<td>78</td>
</tr>
<tr>
<td>$b_{11}$</td>
<td>0.072</td>
<td>0.066</td>
<td>0.080</td>
</tr>
<tr>
<td>$b_{22}$</td>
<td>-0.129</td>
<td>-0.069</td>
<td>-0.088</td>
</tr>
</tbody>
</table>

Laminar: $\delta^* \sim \frac{\delta_{99}}{3}$
Turbulent: $\delta^*$ in log-law region
MARVS Reynolds stress model, source term for $b_{ij}$ equations

\[
S_{bij} = -\frac{1}{2} b_{ij}(c_1 - 2)\omega - \frac{1}{2} b_{ij}(2 + \frac{1}{2} c_6) \frac{P}{k} + \frac{1}{2} \left( c_3 - \frac{4}{3} \right) S_{ij} \\
+ \frac{1}{2} \left( c_4 - 2 \right) (b_{ik} S_{jk} + b_{jk} S_{ik} - \frac{2}{3} \delta_{ij} b_{mn} S_{mn}) + \frac{1}{2} (c_5 - 2) (b_{ik} W_{jk} + b_{jk} W_{ik})
\]

$c_1$ models slow pressure-strain and anisotropy of dissipation. Note term is relatively small when $S/\omega$ is large.

$c_3, c_4, c_5, c_6$ model rapid pressure-strain. $c_5 = 0$. $c_6 = 0$ for shear flow.

M - Moore
A - anisotropy of Reynolds stresses
R - rapidly distorting or fully developed Reynolds stresses
V - vorticity – shear flow or irrotational flow
S - splat wall boundary condition for $b_{ij}$

Data base: Experiments, DNS, rapid distortion theory and theoretical limits. Homogeneous turbulence, with/without vorticity and coordinate rotation. Fully developed pipe and channel flows.
Relationship to linear stability theory and Tollmien-Schlichting waves

Unstable frequencies for a Blasius profile from Schlichting, 1975

MARVS flat plate transition

Vary $Tu_o$ and $R_{t_0}$

$\omega_{f.s.}$ $\omega$ (model) in the free stream.

--- pre-transition trajectories for $Tu_o = 6\%$.

--- start of transition

Linear stability theory considers velocity profiles which are instantaneously inflectional due to the fluctuating velocities. As well as in boundary layers, these also occur in uniform shear flows and are a (hidden) part of homogeneous turbulence modeling, i.e. the MARVS model.
Flat Plate - thin leading edge.

Why does the model work?

Turbulence near walls and in pre-transitional boundary layers is streaky.

This is primarily due to the effect high dimensionless strain rate, $S/\omega$, has on homogeneous turbulence - not the proximity of a wall.

The vorticity in the shear flow, turns the Reynolds stress tensor in the opposite direction more when $S/\omega$ is high.

Turbulence production is reduced due to the poor correlation between the principal directions of the tensors $-u_iu_j$ and $S_{ij}$.

The MARVS model captures this essential homogeneous turbulence physics.
2-d Turbine Cascade Heat Transfer

MARVS model -> flow, $q, \omega$. Use with EVM model, $Pr=0.7$, uniform heat flux

Butler et al. (2001) data laminar using MARVS model velocities
Turbine Cascade Leading Edge

- Static pressure contours
- ~8 points in Hiemenz b.l.
- s-side
- p-side

Graph:
- Nu / \sqrt{Re}
- Data points
- MARVS/EVM laminar
- s/c = 0
- pressure side - suction side
Way Forward - Reynolds Stress Model - Separated Flow

Classic inflectional velocity profile is unstable.

This inhomogeneous effect is NOT included in MARVS development.
Analyze inviscid Kelvin-Helmholtz instability for turbulence (x-average)

\[
L_s = \frac{U_0}{dU/dy_{\text{max}}}
\]

\[
y/L_s = 1 \quad \frac{U_0}{2}
\]

\[
y/L_s = -1
\]

\[
-U_0/2
\]

Exact k-equation balance

\[
\frac{1}{k \frac{\partial U}{\partial y_{\text{max}}}} \left( \frac{\partial k}{\partial t} = -2kb_{12} \frac{\partial U}{\partial y} - \frac{1}{2} \frac{\partial u_iu_iu_2}{\partial y} - \frac{1}{\rho} \frac{\partial pu_2}{\partial y} \right)
\]

\[
\frac{y}{L_s} = 0 \quad 0.38 = 0.86 \quad - \text{small} \quad - 0.48
\]

\[
\left| \frac{y}{L_s} \right| > 1.5 \quad 0.38 = 0 \quad - \text{small} \quad + 0.38
\]

Large pressure diffusion. Outside shear layer ln\(q\) linear in \(y\).
Does data show Kelvin-Helmholtz instability turbulence? **YES!**

![Graph showing data case 106 with Tu=0.6% contours u'/Uo and delta log(u'/Uo)=0.2.](image)

**Kelvin-Helmholtz instability?**

(a) Does log u’ increase linearly in x in center of shear layer? **Yes**

\[
(D \ln u' / Dt) / (dU / dy)_{\text{max}} = 0.19 \quad 0.064
\]

Inviscid 2-d T3L 0.6% data

(b) Does log u’ vary linearly in y outside the shear layer? **Yes**

\[
\nu_t = c_{p.d} U_o^2 / (dU / dy)_{\text{max}} \quad c_{p.d.} = 0.23 \quad 0.08
\]
Preliminary Instability Model: Use search procedure to find instability regions. Cancel out some turning by vorticity with $c_5$ pressure-strain. Add pressure-diffusion of $k$ and $\omega$, but not $b_{ij}$, over larger region.

T3L case $Tu=0.6\%$

DATA

MARVS + Instability model
Kasagi 2:1 Backstep - Rd = 9550, fully developed turbulent inlet

Compare $q/U_{max}$

MARVS

Kasagi Data (1993)

MARVS + Instability model
M4D
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CLASSICAL AND TURBULENT FLOWS

moore64.home.comcast.net/toolbox/m4d.2015.2.tar.gz

Input and sample output for 6 ready-to-run examples.
Resources

Website:  moore64.home.comcast.net

Click on Papers and Presentations  - recent papers etc.

Software

M4D:  moore64.home.comcast.net/toolbox/m4d.2015.2.tar.gz

Flat Plate Transition:  moore64.home.comcast.net/toolbox/flatplate.jgm.tar

Book:  Functional Reynolds Stress Modeling
Joan G. Moore and John Moore, Oct. 2006

Available from:
Amazon.com
In several libraries
M4D - Documentation

What you can do:
Compile the C code (see m4d.whatshere.setup.run.pdf) then

Try the given examples - see doc.rtf for each example

Modify the examples or make up your own examples -
see m4d.commands.pdf and m4d.commands.short.pdf

For the programmer:
Add commands to M4D to extend its capabilities -
see m4d.subroutines.pdf and m4d.variables.pdf
M4D Input - Command Format

---

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>c: comment ****** box cavity: convection around 180 degrees (half vortex)</td>
<td></td>
</tr>
<tr>
<td>c: constant alim d 1 .1</td>
<td>control volume limit parameter</td>
</tr>
<tr>
<td>c: constant dt d 1 .1</td>
<td>time step</td>
</tr>
<tr>
<td>c: infile inn.grid11x6 1</td>
<td>set up grid</td>
</tr>
<tr>
<td>c: infile inn.init 1</td>
<td>initialize variables</td>
</tr>
<tr>
<td>c: infile inn.init.coefLL 1</td>
<td>set up control volumes and coefs</td>
</tr>
</tbody>
</table>

--- setups and initial plots ---

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>c: constant p.defaultdir s 1 ../../.jgm</td>
<td>default dir for color.map etc</td>
</tr>
<tr>
<td>c: arraydump out/converge alim dt &quot;&quot;</td>
<td>start convergence file</td>
</tr>
<tr>
<td>c: infile inn.plotcv 1</td>
<td>plot control volumes out/cv0.gif</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>c: constant ITERPLOT i 1 1</td>
<td>time steps between cc plots</td>
</tr>
</tbody>
</table>

------------ start calculation ------------

<table>
<thead>
<tr>
<th>Command</th>
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</tr>
</thead>
<tbody>
<tr>
<td>c: infile inn.step.o2c 50</td>
<td>take 50 2nd-order time steps</td>
</tr>
<tr>
<td>c: arraydump out/cc50 cc &quot;&quot;</td>
<td>save concentration on file</td>
</tr>
</tbody>
</table>
M4D Differential Equations

Continuity
\[ \frac{\partial \rho U_i}{\partial x_i} = 0 \] steady or incompressible.

Momentum - inviscid, laminar or turbulent, steady or unsteady. Select needed terms.

\[ \rho \frac{\partial U_i}{\partial t} + \rho U_k \frac{\partial U_i}{\partial x_k} - \frac{\partial}{\partial x_k} \mu \frac{\partial U_i}{\partial x_k} - \frac{\partial}{\partial x_k} \mu_t \frac{\partial U_i}{\partial x_k} = \]

\[ - \frac{\partial \rho u_i u_k}{\partial x_k} - \frac{\partial p}{\partial x_i} - \delta_{i1} \frac{\partial p}{\partial x} - 2 \rho \varepsilon_{i3m} \Omega^2 U_m + \rho \frac{\partial}{\partial x_i} \left( \frac{\Omega^2 r^2}{2} \right) + \frac{\partial}{\partial x_k} (\mu + \mu_t) \frac{\partial U_k}{\partial x_i} \]

\( \rho, U_i \) at grid points

Continuity control volumes, \( \rho U_i \) 3d linear

Momentum control volumes, \( U_i \) 3d linear

\( p \) at corners of momentum control volumes
one point in each continuity control volume

Pressure-correction solution procedure
M4D Differential Equations cont.

Turbulence model properties, generic form. Select needed terms.

\[ \rho \frac{\partial \phi}{\partial t} + \rho U_i \frac{\partial \phi}{\partial x_i} - \frac{\partial}{\partial x_i} \mu \frac{\partial \phi}{\partial x_i} - \frac{\partial}{\partial x_i} \mu_t \frac{\partial \phi}{\partial x_i} - \frac{\partial}{\partial x_i} \mu_{ik} \frac{\partial \phi}{\partial x_k} = S_\phi \]

Models:
Reynolds stress (MARV or MARVS)
2 Eq. (Coakley with modified \( \omega \) bndry. condition)

\[ \phi = q, \omega, b_{ij} \]

Conserved species, including energy. Select needed terms.

\[ \rho \frac{\partial \phi}{\partial t} + \rho U_i \frac{\partial \phi}{\partial x_i} - \frac{\partial}{\partial x_i} \mu \frac{\partial \phi}{\partial x_i} - \frac{\partial}{\partial x_i} \mu_t \frac{\partial \phi}{\partial x_i} - \frac{\partial}{\partial x_i} \mu_{ik} \frac{\partial \phi}{\partial x_k} = 0 \]

Model: MARV/MARVS-compatible EVM for \( \mu_t / \text{Pr}_t \) using \( q, \omega, \text{Pr} \)

Use convection adapted control volumes, like for momentum.
M4D Imbedded Plot Package

Color fill contours, velocity vectors, and/or grid lines.

*Plot each timestep to create frames for videos.* (.gif files)

Labeled color bars.

Line plots - linear or log scales

Control volumes.
M4D
A RESEARCH CFD CODE
FOR THE CALCULATION OF
CLASSICAL AND TURBULENT FLOWS

moore64.home.comcast.net/toolbox/m4d.2015.2.tar.gz

Input and sample output for 6 ready-to-run examples.
MARVS Reynolds stress model tests:

2-D Fully Developed Rotating Channel - steady turbulent flow, $U_x$ momentum and Reynolds stress equations.

Compare $U/U_m$ M4D MARVS with MARV and KA1993 DNS.
M4D - input commands -> procedure:
- Time accurate or steady
- Inviscid, laminar or turbulent
- Incompressible - pressure correction method

Paradigm shift features:
- 3-d linear profiles over convection adapted control volumes

MARVS Reynolds stress model transition - shear flow at high $S/\omega$