DISTRIBUTED VORTEX WAVE INTERACTION ARRAYS AND TURBULENT SHEAR FLOWS

PHILIP HALL, MONASH UNIVERSITY,

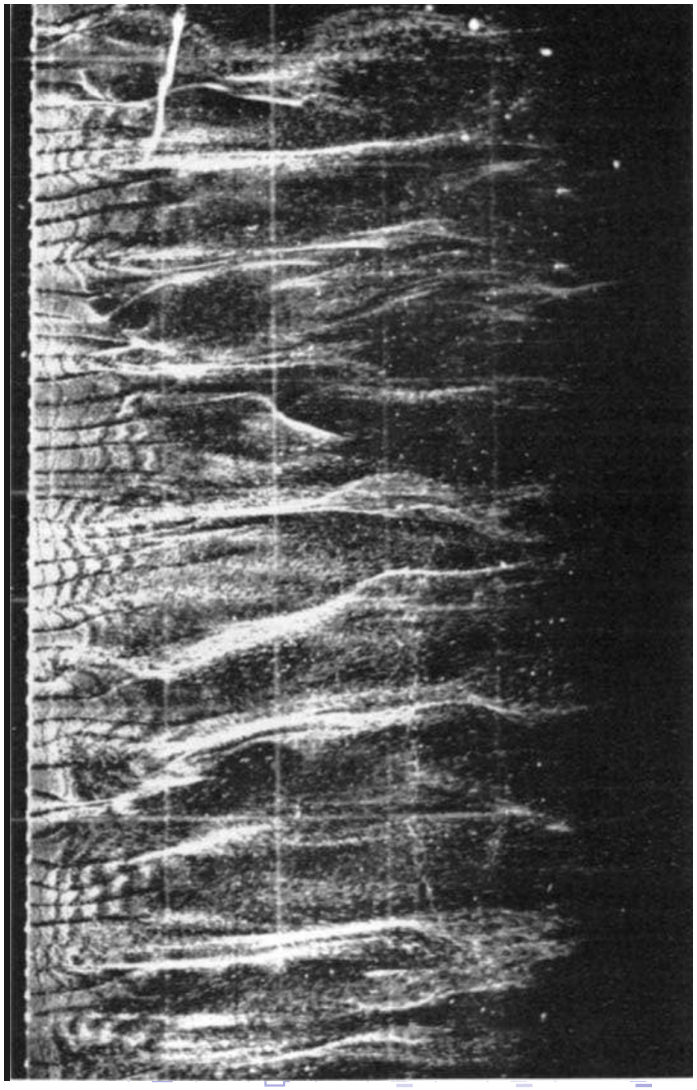
August 5, 2019

SUMMARY

- **BACKGROUND**
  Key properties of turbulent shear flows.
- **EXACT COHERENT STRUCTURES**
  Relationship to transition and turbulence.
- **VWI**
  The sustaining mechanism for turbulence.
- **SELF SIMILARITY IN TURBULENT SHEAR FLOWS**
  VWI arrays, the law of the wall, uniform momentum zones.
- **CONTROL OF TURBULENCE**
  How to manipulate ECS?

EXPERIMENTAL RESULTS OF KLINE ET AL (1967)

- Flow from left to right
- Smoke released into airflow near leading edge
- Spanwise periodicity associated with streaks revealed



THE LAYERS OF WALL-BOUNDED TURBULENCE

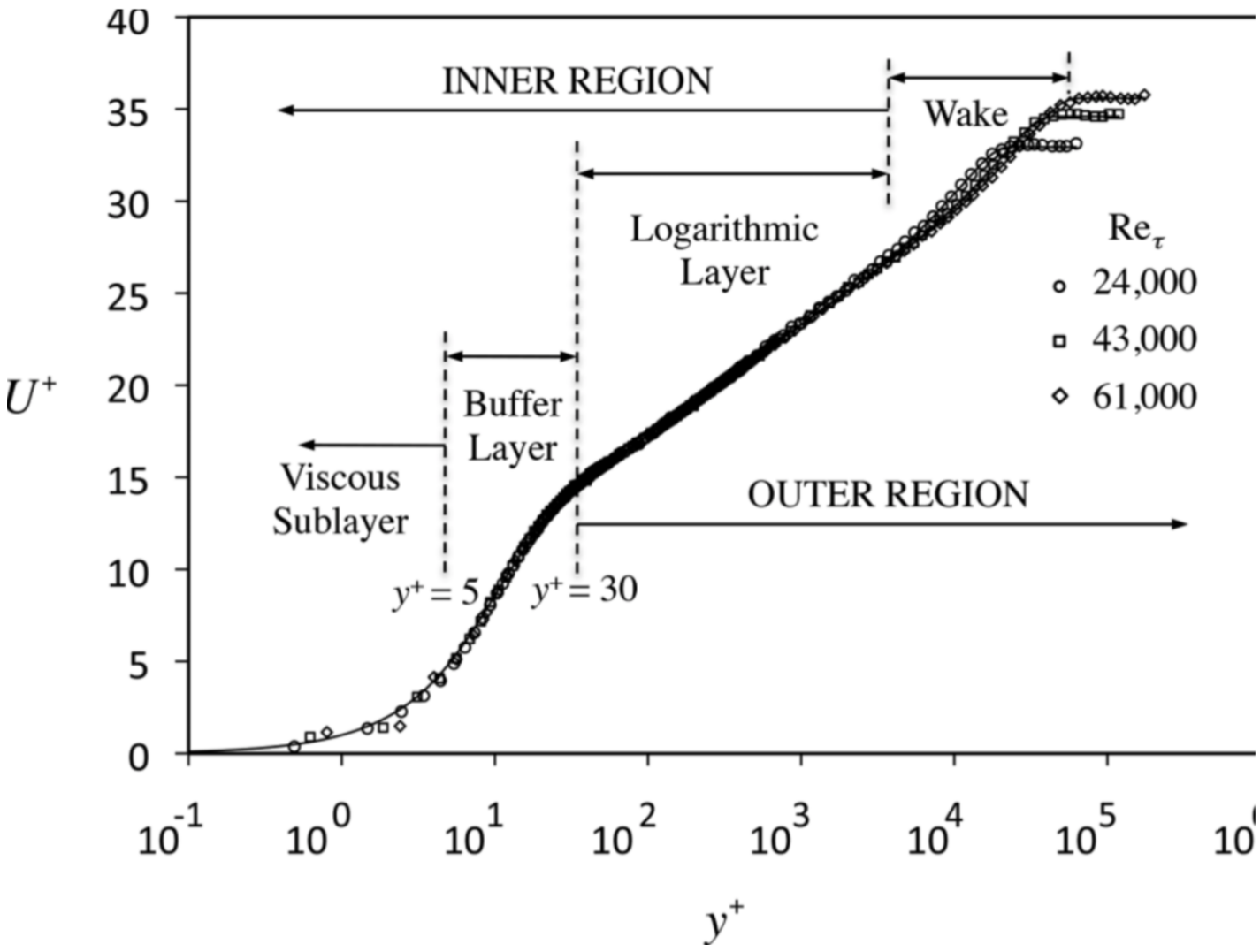


Figure: In log layer $U^+ = \frac{1}{\kappa} log y^+ + B$

UNIFORM MOMENTUM ZONES (Hutchings et al 2013)

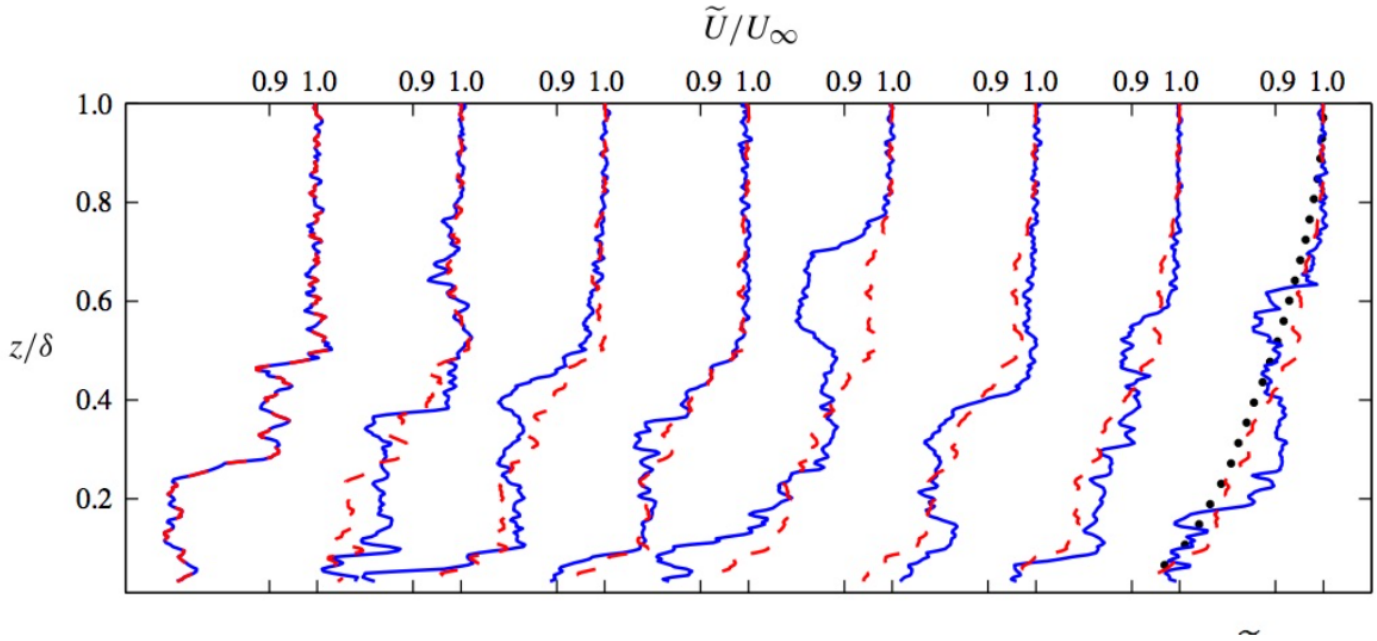


Figure 2: Instantaneous streamwise velocity in a turbulent shear flow as a function of distance from the wall. The horizontal axis denotes the streamwise velocity divided by its free stream value and the vertical axis distance from the wall scaled on boundary layer thickness.

EXPT. SHOWING LINEAR INCREASE OF λ_z, λ_x

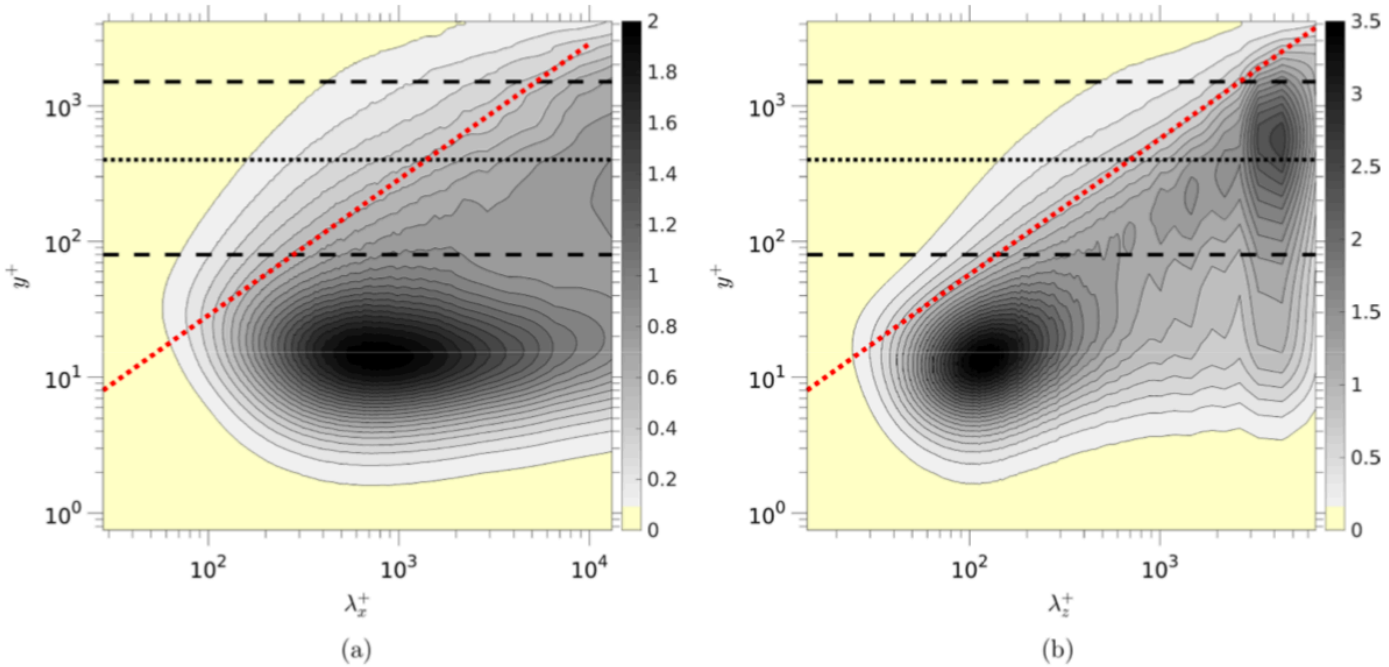
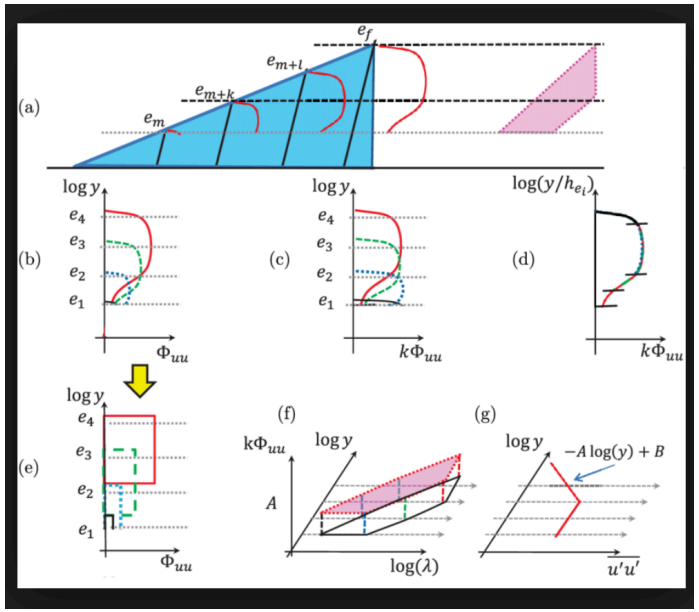


FIG. 2. Premultiplied power spectrum of the streamwise fluctuations in both the (a) streamwise and (b) spanwise directions at $\mathrm{Re}_\tau = 4200$. The dotted red lines show either the relation $\lambda_x^+ = 3.5y^+$ or $\lambda_z^+ = 7y^+ = 2\lambda_x^+$.

TOWNSEND'S ATTACHED EDDY HYPOTHESIS

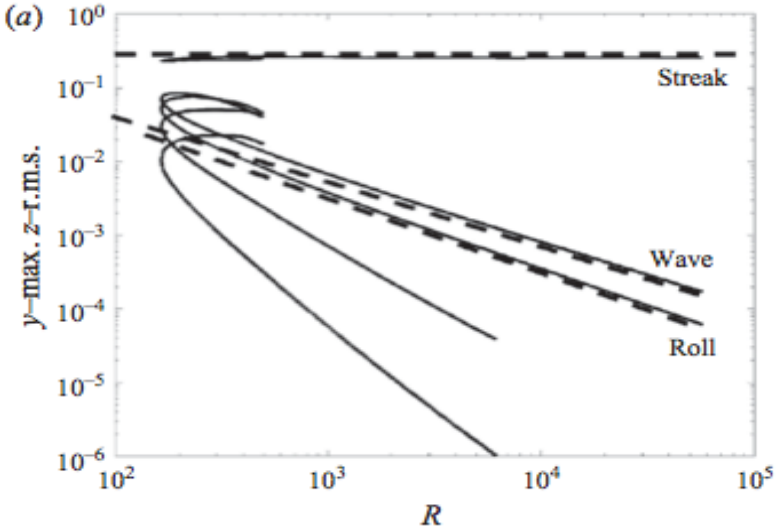
- Wavelengths scale with distance from the wall
- Assumes a generic structure in outer part of the log layer
- Hypothesis has limited 'physics' input

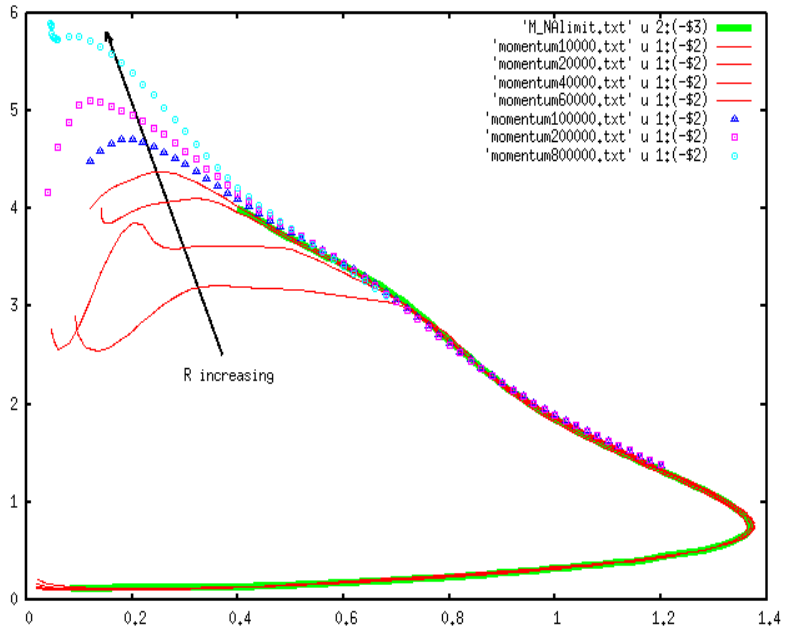


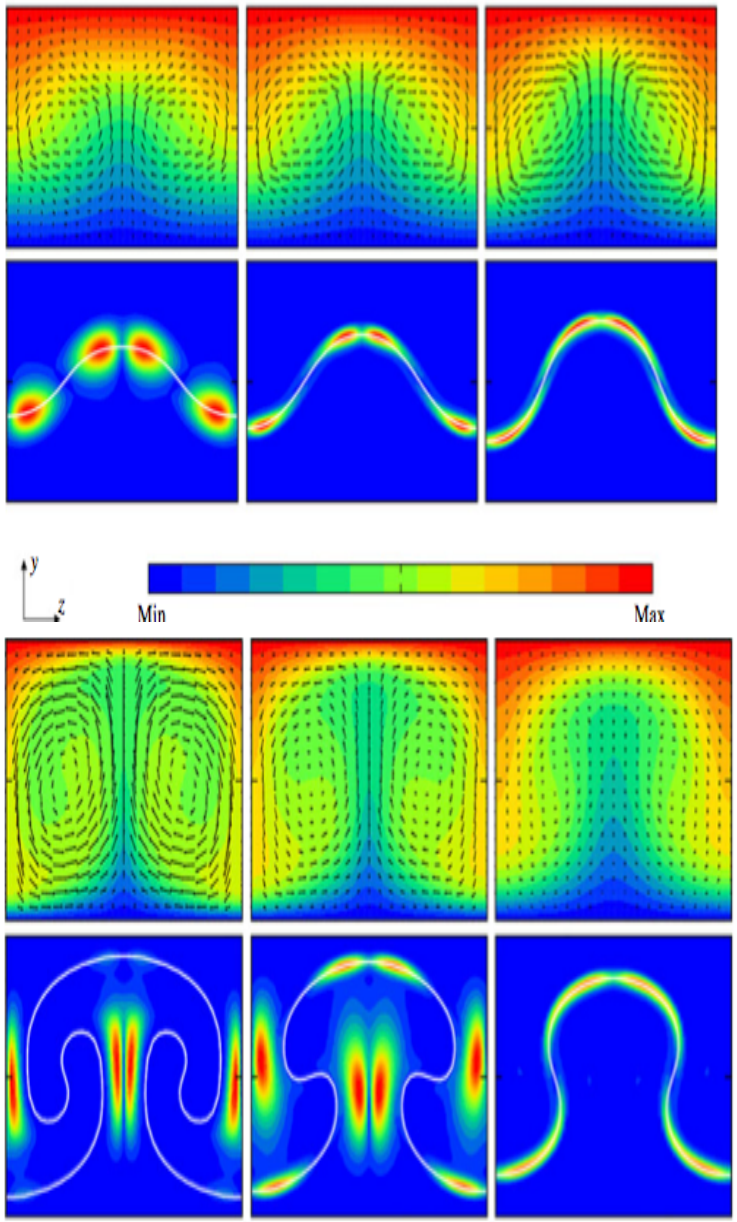
WHAT ARE EXACT COHERENT STRUCTURES?

- 3D nonlinear unstable solutions of the Navier Stokes equations.
- First solutions given by Nagata (1990) for Taylor Couette flow.
- Waleffe and collaborators (late 1990's) rebrand as self-sustained processes-SSP.
- Kerswell (2000+) pipe flow equilibrium sols.
- Solutions appear as a saddle node bifurcation at finite R and form upper and lower branches.
- Lower branch solutions define the 'edge' and describe bypass transition. Upper branch modes are more energetic, more unstable and are local attractors for turbulent simulations.
- ECS made up of streaks (spanwise corrugated mean flow in x direction), rolls (O(1/R) smaller) in y-z plane and 3D waves travelling in x direction.
- Waleffe rediscovered the sustaining mechanism by which streaks unstable to waves which drive rolls which in turn drive the streak.

COUETTE ECS: LOWER & UPPER BRANCHES

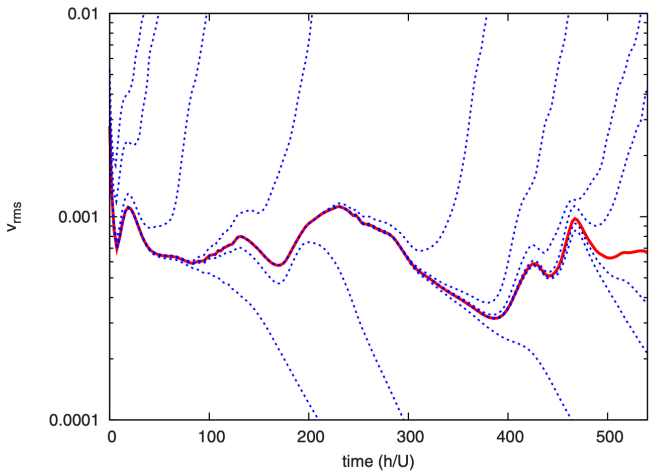






RELEVANCE LOWER BRANCH TO TURBULENCE (1)

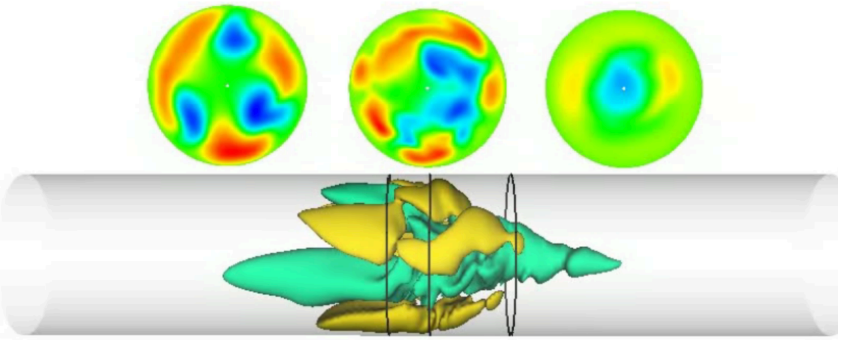
- Lower branch states have only one unstable direction.
- DNS simulations from initial disturbance-refine initial amplitude.
- 'Edge' state is limit of refinement, (Skufca, Yorke 2003).
- Edge state related to bypass transition, optimize to find minimal seed to produce turbulence.



RELEVANCE LOWER BRANCH TO TURBULENCE (2)

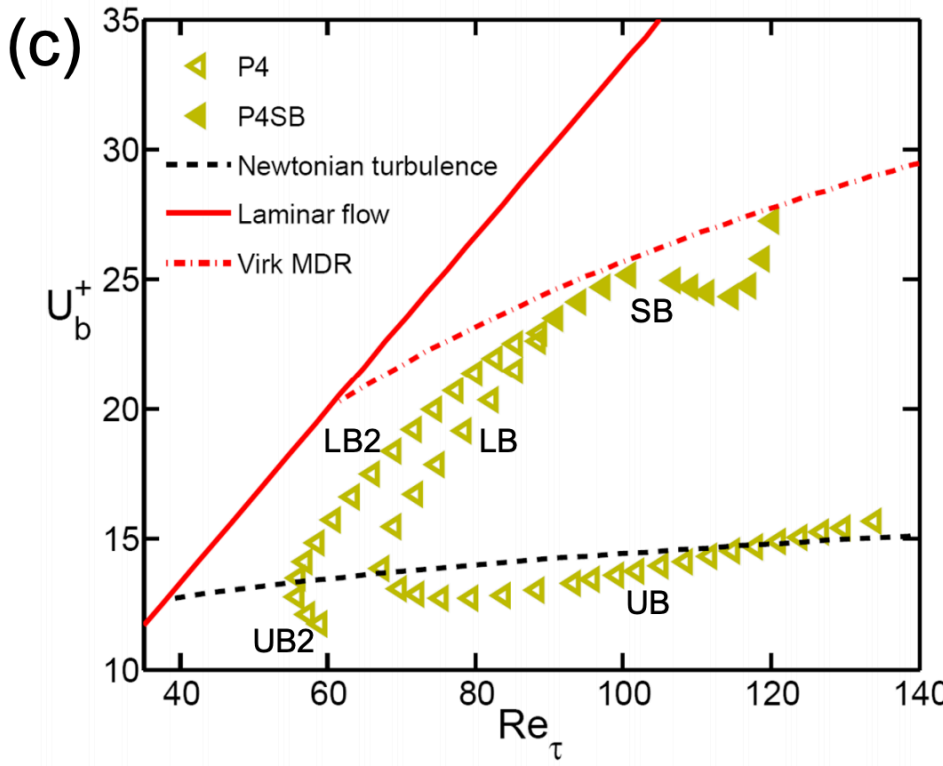
- Refine localized initial disturbance using edge-tracking.
- Approaches 'spotlike' solutions.
- Spot edge state similar to Mullin's experimental results..
- Edge state related to bypass transition, optimize to find minimal seed to produce turbulence.

Localized edge state in pipe flow



RELEVANCE LOWER BRANCH TO TURBULENCE (3)

- Lower branch has much lower drag than upper one- and only one unstable mode.
- Numerical simulations show polymers can significantly reduce drag - optimum is the so-called Virk minimum drag asymptote
- Numerical solutions by Mike Graham(Wisconsin) show the above is the lower branch state



DNS TURBULENT SIMULATIONS AND ECS

- Results from Gibson et al (2009)

Visualizing the geometry of state space in plane Couette flow 121

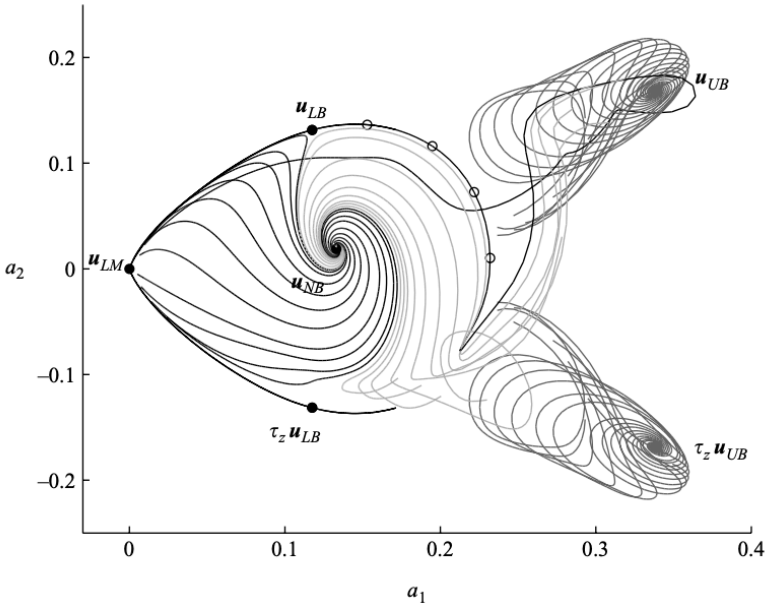


FIGURE 5. A state-space portrait of plane Couette flow for $Re = 400$ and $[L_x, L_y, L_z] = [2\pi/1.14, 2, 4\pi/5]$, projected from 61 506 dimensions to 2. The labelled points are exact equilibrium (steady-state) solutions of the Navier–Stokes equation (see §3); the curved trajectories are fully resolved time-dependent numerical integrations of Navier–Stokes projected onto the $\boldsymbol{e}_1, \boldsymbol{e}_2$ plane defined by (4.3), with the projection $a_n = (\boldsymbol{u}, \boldsymbol{e}_n)$ defined by (4.1). The laminar equilibrium is $\boldsymbol{u}_{LM}$; the equilibria labelled $\boldsymbol{u}_{LB}$, $\boldsymbol{u}_{NB}$ and $\boldsymbol{u}_{UB}$ are shown in figure 3. W^u_{LB}, the one-dimensional unstable manifold of the 'lower-branch' equilibrium $\boldsymbol{u}_{LB}$, and $\tau_z W^u_{LB}$, its half-cell translation in z, are shown with thick black lines. $W^{u(1,2)}_{NB}$, a two-dimensional portion of the unstable manifold of $\boldsymbol{u}_{NB}$, is shown with thin black and grey spirals emanating from $\boldsymbol{u}_{NB}$. Similarly, the dark grey lines spiralling out of $\boldsymbol{u}_{UB}$ and $\tau_z \boldsymbol{u}_{UB}$ indicate $W^{u,S}_{UB}$ and $\tau_z W^{u,S}_{UB}$, the two-dimensional unstable manifolds of $\boldsymbol{u}_{UB}$ and its half-cell translation $\tau_z \boldsymbol{u}_{UB}$. Open dots along W^u_{LB} show initial conditions for Newton-GMRES searches used to find $\boldsymbol{u}_{NB}$. The plane of the projection is defined in terms of the equilibrium solutions; it is dynamically invariant and independent of the numerical representation. See §§4.2, 4.3 and 3.2 for discussions of the projection, dynamics and numerical discretization.

A BRIEF HISTORY OF THE ORIGINS OF VWI

- Steady Streaming theory of Stuart (1966), Reynolds stresses.
- Malkus 'Marginal Theory of Turbulence' 1956.
- Mean flow generation by small 3D waves -Benney 1967.
- Andrews and Mcintyre (1976) , Generalized Lagrangian Mean' description of waves.
- Hall-Smith 88-91 VWI theory of waves and vortices. Independent of Benney's work but gives precise structure consistent with Benney's speculation.
- SSP numerical calculations of ECS Waleffe 1997, Wedin and Kerswell (2003),.....∞.
- VWI can be seen as the local sustaining mechanism for turbulence

THE VWI STRUCTURE

- Consider any flow $u(y, z)$ in x direction: streak flow.
- Couples to a weak O(1/R) roll flow perpendicular to streak.
- Consider inviscid instability wave riding on u staifies generalized Rayleigh equation:

$$\mathrm{div}\left(\frac{\nabla P}{[u-c]^2}\right) = 0, \nabla P \circ \mathbf{n} = 0, \text{at boundary.}$$

- Singular where $u = c$, viscous critical layer, acts like a wave guide.
- Increase wave size until steady streaming gives $O(1)$ jump in roll stress.
- Now wave drives roll which drives streak: asymptotic closure.
- Mechanism rediscovered by numerical interrogation by Waleffe 1997 and renamed SSP- self-sustained process.

THE VWI MECHANISM

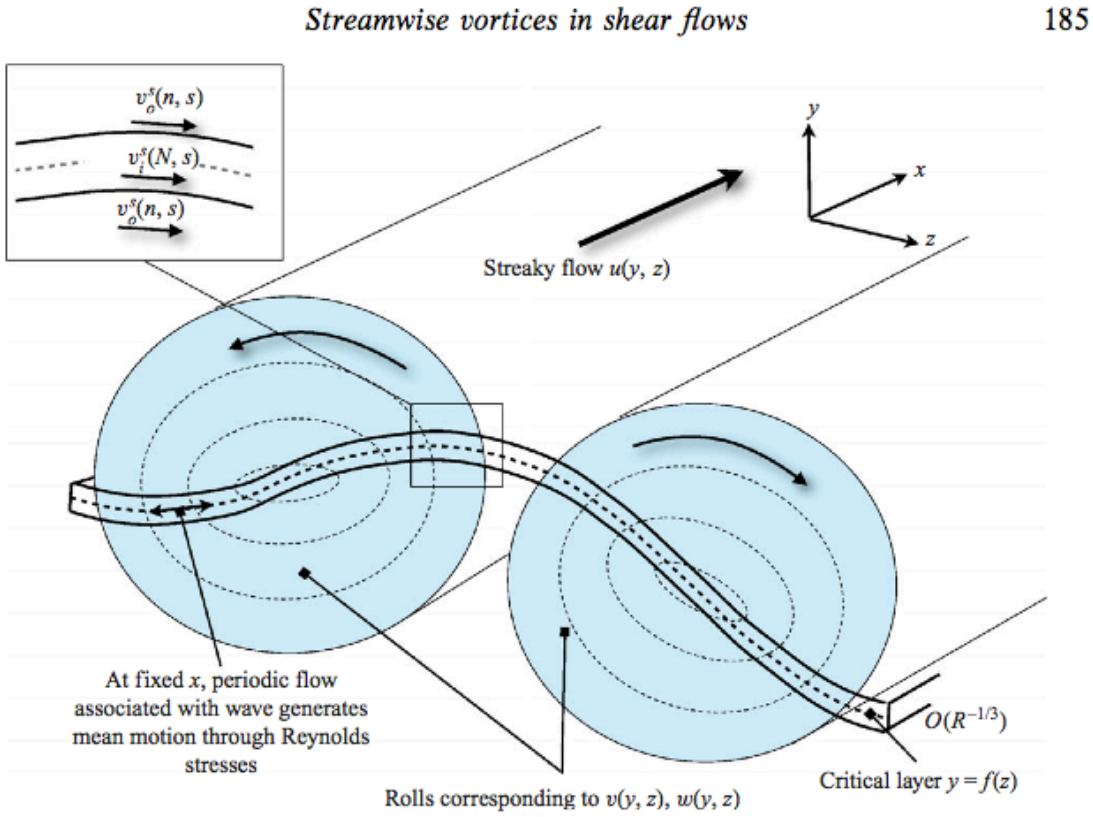


Figure: Wave exists as instability of streak, its associated Reynolds stresses drive the roll flow in its critical layer, roll flow drives the streak,.....

Take Navier Stokes in the form:

$$\mathbf{u}_t + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \frac{1}{R}\nabla^2\mathbf{u},$$
$$\nabla \cdot \mathbf{u} = \mathbf{0}$$

THE TIMESCALES FOR VWI:

1. $t = O(R^{-1})$ the slow roll-streak diffusion timescale.
2. $t = O(1)$ the fast inviscid wave timescale
3. $t = O(R^{-1/2})$ the 'birth/death' timescale for VWI states.

THE FORMAL ASYMPTOTIC SOLUTION FOR VWI

Away from the critical layer the roll-streak-wave flow is given by

$$\widetilde{u} = (U, R^{-1}V, R^{-1}W) + \cdots + R^{-\frac{7}{6}}(\mathbf{u}E + CC) + \cdots \quad (1)$$

$$\widetilde{p} = R^{-2}P + \cdots + R^{-\frac{7}{6}}(pE + CC) + \cdots , \quad (2)$$

- Here $E = exp[i\alpha(x - ct)]$.
- Roll-streak flow has U, V, etc functions of y, z, $T = \frac{t}{R}$.
- Wave lives on the fast t timescale.
- Critical layer occurs where $U(T, y = f(z, T) = c$, and wave velocity becomes singular like $(y - f)^{-1}$.
- Viscous critical needed to regularise, provides explicit roll stress jumps.

THE VWI INTERACTION EQUATIONS

Simplest case has roll-streak flow steady, more general time-periodic states exist.

Roll/streak problem

$$\mathbf{U} \circ \nabla \mathbf{U} = (0, P_Y, P_Z)^T + \nabla^2 \mathbf{U}, \nabla \circ \mathbf{U} = 0, \nabla = (0, \partial_Y, \partial_Z).$$

$$\mathbf{U} = \pm 1, Y = \pm 1, \mathbf{U}(Z + 2\pi/\beta) = \mathbf{U}(Z)$$

$$[\partial_n[(V, W).\mathbf{s}]_-^+ = J(P_0(f, Z)), [P_0]_-^+ = K(P(f, Z))$$

Inviscid wave problem

$$P_{0YY} + P_{0ZZ} - \frac{2P_{0Y}U_{0Y}}{U - c} - \frac{2P_{0Z}U_{0Z}}{U - c} - \alpha^2 P_0 = 0.$$

Stress jumps J, K functions of wave pressure. Defines a nonlinear eigenvalue problem for wave speed , wave amplitude as functions of wave numbers α, β.

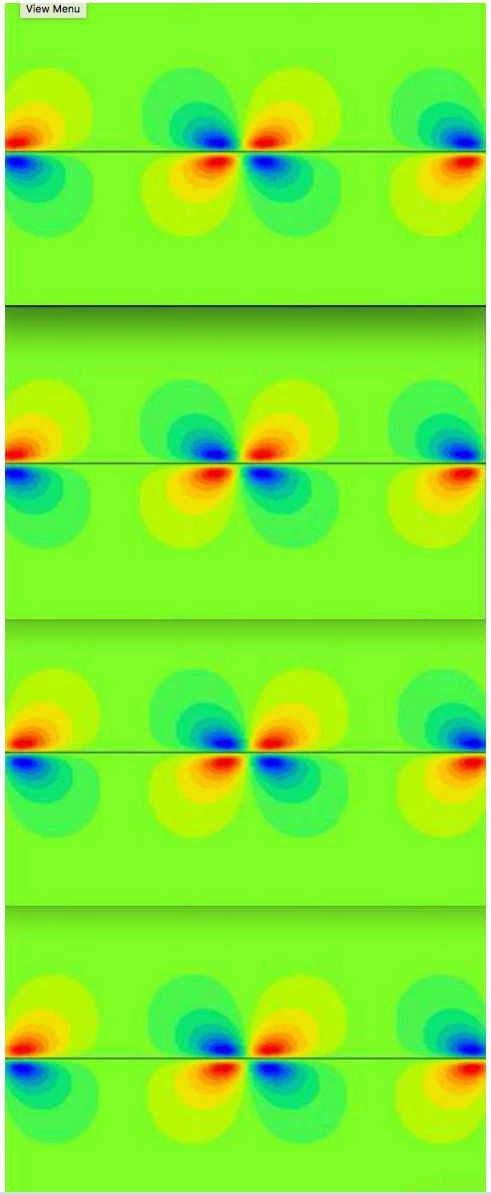
SUMMARY OF VWI RESULTS FOR CHANNEL FLOWS

- VWI describes all known SSP solutions found from full Navier Stokes even for R as low as 10^3.
- Lower branch state has 1 unstable disturbance, upper branch state has many.
- Kawahara-Kida periodic states bifurcate from VWI states, one described by VWI the other by unsteady critical layer theory.
- Lower branch states have lower drag than fully developed turbulence, trapping flow in that state can be achieved by polymers- (Mike Graham, U. Wisc.)
- Results for channels relevant to near wall streaks-essentially need small number of vortices.
- But if we start with Couette flow and let spanwise wavelength shrink discover states for arbitrary shear flows- BHS (2013)
- So now take infinite array of disconnected states in wall normal direction and bring them together.

PERIODIC VWI ARRAYS IN UNIFORM SHEAR FLOW

- Key point is a single VWI produces only a small correction to a linear shear profile.
- Now consider infinite set of canonical localised VWI states each driven by wave locked onto local mean velocity.
- Hall 2018 gave first solutions for such VWI states in a uniform shear flow, vertical wavelength constant. c.f. Sekimoto, Dong, Jimenez 2016

.



PERIODIC ARRAYS OF VWI STATES: HALL 2018

- Consider infinite array of cells $n < y < n+1$, n an integer with unperturbed state $u = y$.
- Assume roll-streak flow (without uniform shear) is identical in each box.
- Now assume an infinite number of waves travelling with speeds $c + n$ so one critical layer in each box and so one wave drives the local roll flow.
- Hence roll and streak are periodic in y direction with infinite set of waves with wavespeed $c + n, n = 0, \pm 1, \pm 2, \pm 3, ...$
- Effect of wave in neighbouring boxes is second order effect.
- VWI mechanism in each cell sustains local cell BUT each wave is not periodic in y.
- Now possible structures depend on how disconnected states are brought together.
- Significant states are 'chevron' and 'varicose' - correspond to initial phase difference of 0 or π.
- Chevron mode similar to LES calculations of Sekimoto, Jimenez (2016). Varicose upper branch mode gives uniform momentum zones.

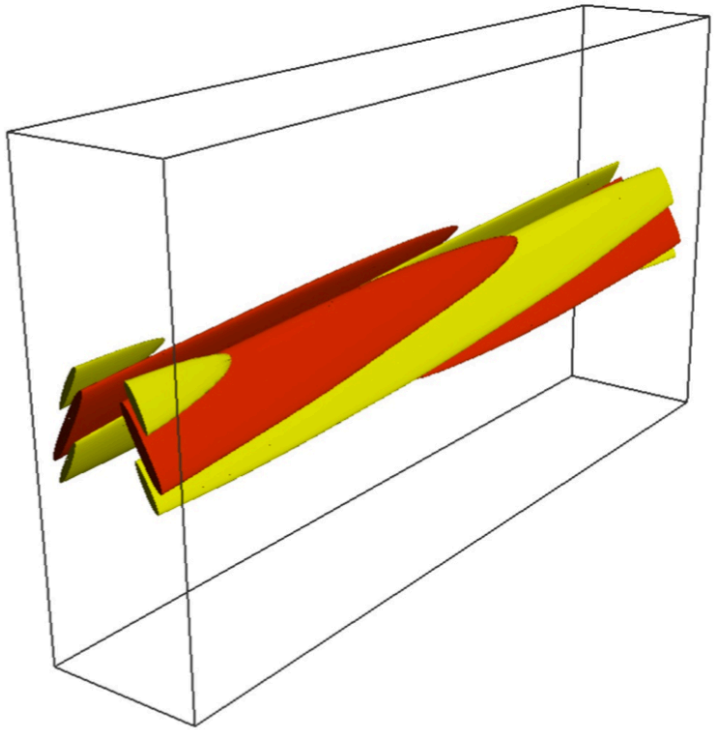
THE CHEVRON MODE

10

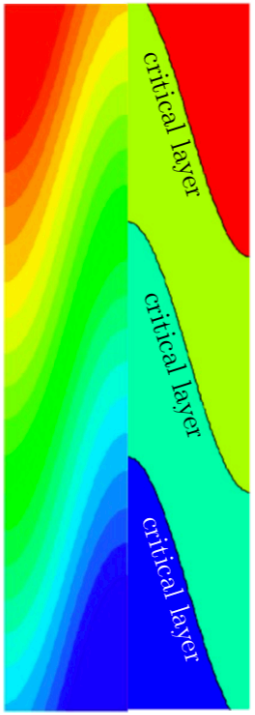
Triply periodic VWI – tilted critical layer branch

Solution for $\alpha = 0.5531$, $\beta = 3$, $\rho = 0.0488$, $Re = 2000$.

Wave streamwise vorticity

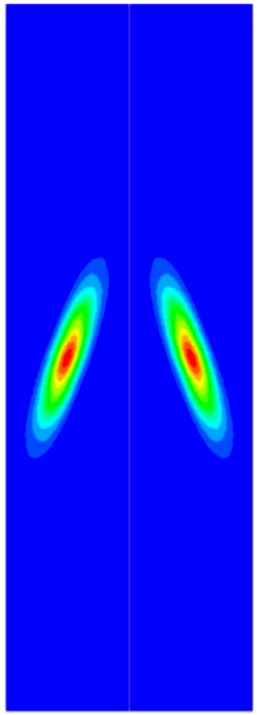


Wave base



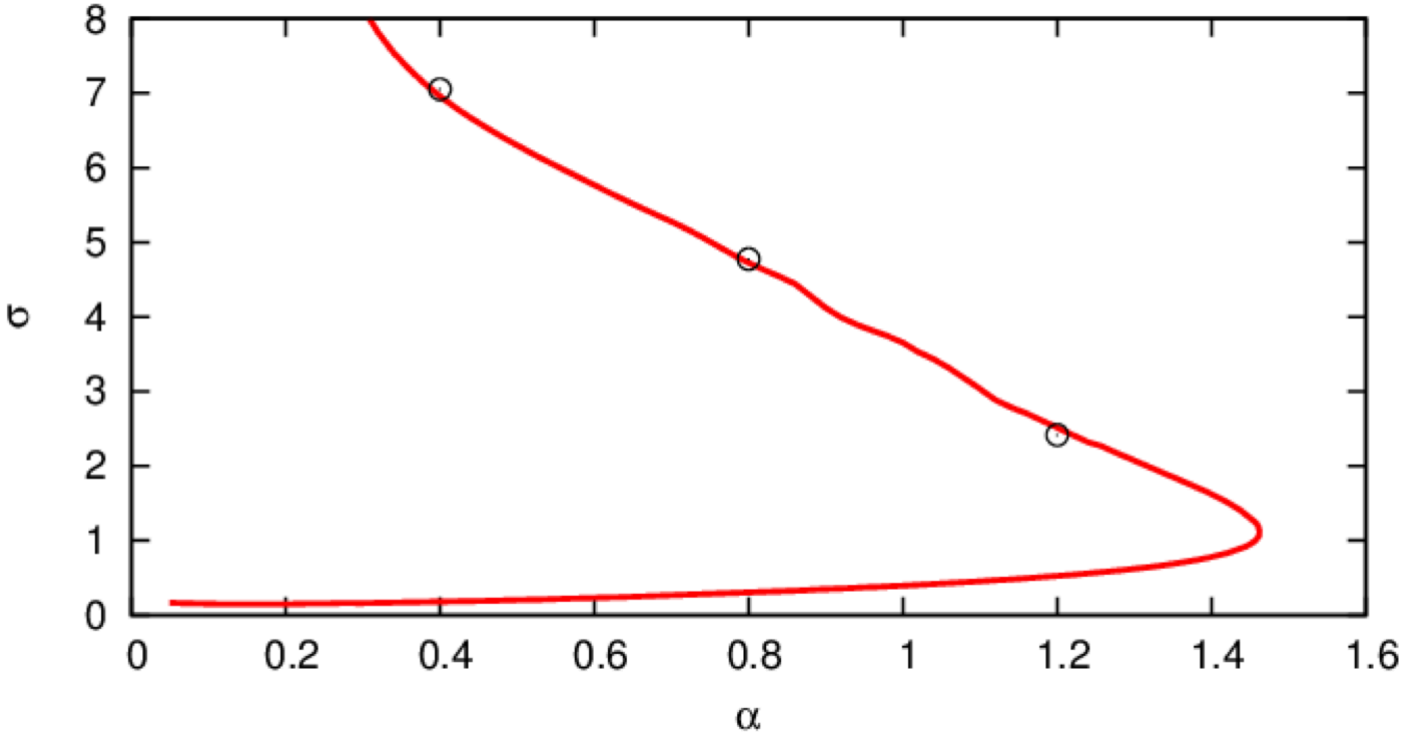
$y + U$

Wave mode KE



$|\widehat{\boldsymbol{u}}|^2$

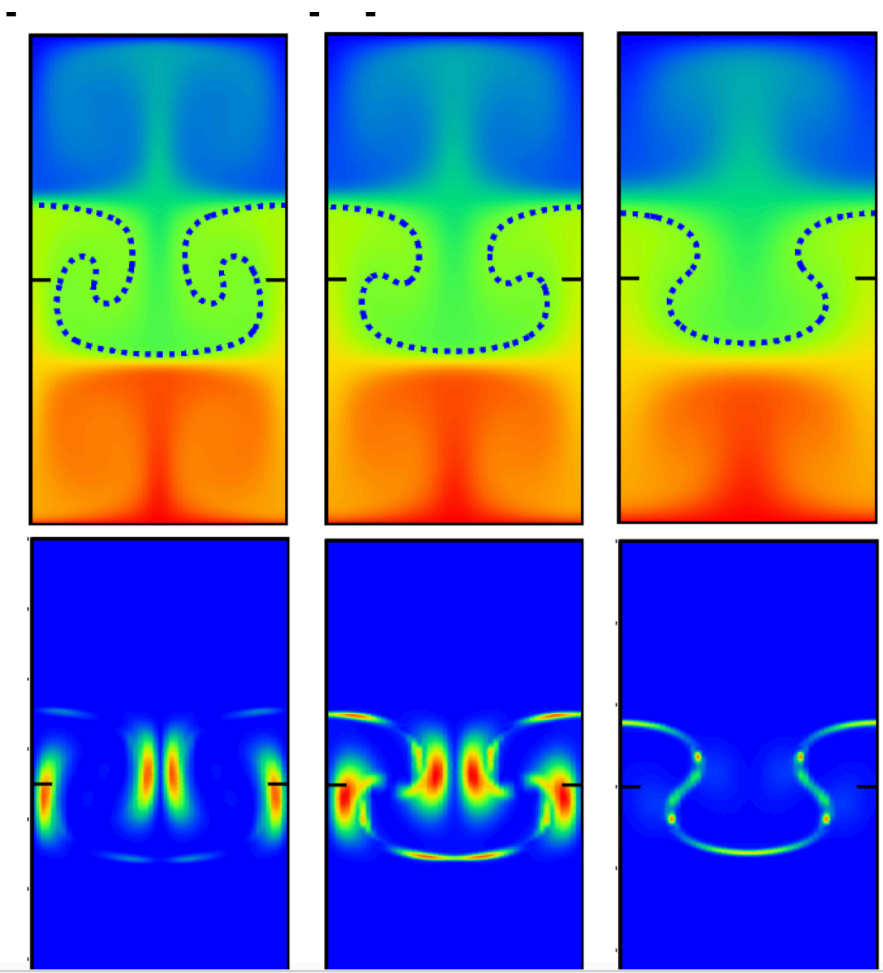
THE VARICOSE MODE



THE VARICOSE MODE

Streak, including the base flow (Note: base flow is –y so look the figure from opposite direction)

Wave KE, namely x average of the u^2+v^2+w^2 for the wave



R=70000 alpha= 0.4,0.8,1.2 from left to right

SUMMARY OF RESULTS FOR PERIODIC ARRAYS

- ECS with both flat and wavy critical layers exist- but wavy ones most energetic.
- ECS develop uniform momentum zones on upper branches.
- So a uniform shear flow can support stacked cells of identical roll-streaks driven by wave with correct local wavespeed.
- Results vaguely similar to expts/DNS in region close to wall of turbulent flow.
- But x, z wavelengths independent of y and base flow $u = y$ specified.
- Expts/DNS suggest wavelengths vary and mean state determined by turbulent fluctuations.
- Now we use Ray Theory to extend to describe the latter situation.

SLOWLY VARYING PERIODIC ARRAYS

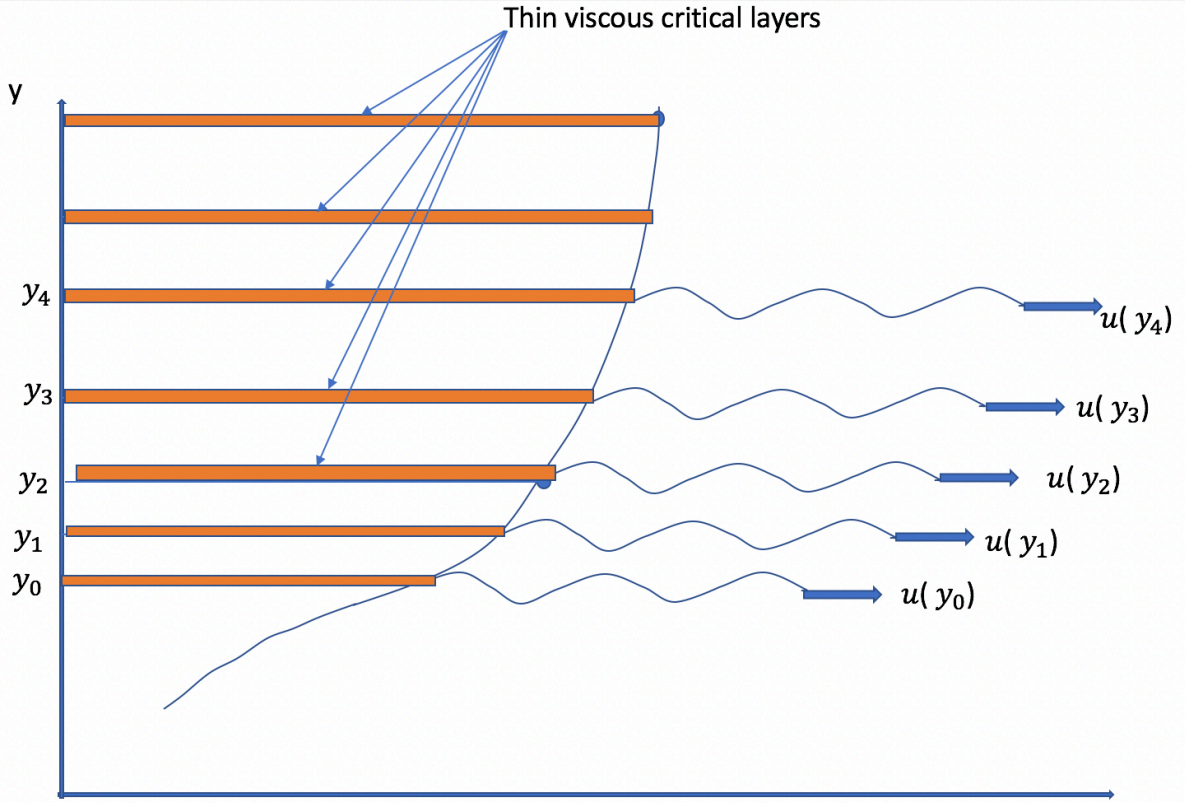
- But spanwise and streamwise wavelengths are constant, whereas experiments show dominant wavelengths vary in y.
- Now use 'Ray Theory' to allow slow variation of states in y.
- But spanwise wavelength must stay constant in y!!!!
- Local depth of cell varies slowly as does streamwise wavelength of the sustaining wave.
- Now work in wall units so take Navier Stokes equations with Reynolds number unity. Large parmeter is now distance from the wall.
- Assume vertical extent of generic structure is of depth ϵ^{-1} so take $Y = \epsilon y$ as slow scale.
- ϵ to be inferred from DNS.
- Local cell wavenumber in Y is $\Theta'(Y)$ - Θ is phase function.

So define phase variable Φ in y direction by $\Phi = \frac{\theta(Y)}{\epsilon^{\frac{2}{5}}}$.

(2/5 is VWI scaling within wall units)

SLOWLY VARYING PERIODIC ARRAYS

- Choose infinite set of waves centred on $[y_n]$ with wave speed $\overline{u}(y_n)$.
- Waves not equally spaced in y , each one sustains roll in its own cell- but second order effect on neighbouring cells.
- So have periodicity in phase variable, wave frequencies slowly changing. (But different waves so OK).
- Phase function $\theta(Y)$ is to be found.



DISTRIBUTED ECS IN A RAY THEORY FRAMEWORK

- Consider waves of spanwise/streamwise/vertical wavelength $O(\epsilon^{-3/5})$.
- Hence define $(X, Z) = \epsilon^{\frac{3}{5}}(x, \beta z)$.
- Now seek a solution using ray theory and expand

$$\epsilon u = \overline{u}(Y) + \frac{\overline{u}'}{\Theta'}[\epsilon^{\frac{2}{5}} U_1(\Phi, z, Y) + \epsilon^{\frac{4}{5}} U_2(\Phi, z, Y) + ...] + u_w, \quad (3)$$

$$\frac{v}{\epsilon} = \Theta'[\epsilon^{-\frac{2}{5}} V_1(\Phi, Z, y) + V_2(\Phi, Z, y) + ...] + v_w, \quad (4)$$

$$\frac{w}{\epsilon} = \Theta'[\epsilon^{-\frac{2}{5}} W_1(\Phi, Z, y) + W_2(\Phi, Z, y) + ...] + w_w, \quad (5)$$

$$p = -\Gamma\overline{p}(x) + \Theta'^2\overline{u}'[\epsilon^{\frac{8}{5}} P_1(\Phi, Z, y) + \epsilon^2 P_2(\Phi, Z, y) + ...] + p_w. \quad (6)$$

And the wave pressure

$$p_w = \epsilon^{\frac{4}{5}} \rho_n [\overline{u}'\Theta'(Y)|^{\frac{5}{6}} \sum_{-\infty}^{\infty} [e^{i\alpha[X-\overline{u}(y_n)T]} \widetilde{p}_{1n}(\Phi - \Phi_n, Y, Z) + C.C. + O(\epsilon^{\frac{2}{5}})]. \quad (7)$$

SLOW DYNAMICS : DETERMINATION OF $\overline{u}, \theta', \alpha$.

We confine attention here to the zero mean pressure gradient case.

- At this stage $\theta'(Y), \bar{u}$ and local wavenumber $\alpha(Y)$ are all unknown.
- Leading order problem = infinite periodic problem in a linear shear but effective local wavenumbers $\alpha/\theta', \beta/\theta'$ vary in Y.
- So locally have periodic (in Y) structure, but properties vary slowly.
- At next order in ϵ two solvability conditions have to be satisfied for a solution to exist

$$\overline{u}'' + [\sigma(\Theta')\overline{u}]' = 0. \implies \overline{u}' = \frac{1}{1+\sigma}$$

$$[\frac{\alpha}{\Theta'}]' + q(\alpha, \Theta, \eta).$$

Here σ is a function of the slow variable Y through its dependence on $\hat{\alpha}, \hat{\beta}$ and is defined by

$$\sigma = \sigma(\hat{\alpha}, \hat{\beta}, Y) = -\frac{1}{2\pi}\int_0^1 \int_0^{2\pi} UVdZd\Phi, \tag{8}$$

So we have an explicit closure of the RANS equation for the mean!! The second equation fixes the wavelength $\alpha(Y)$ distribution needed to sustain the interaction- q a complicated function involving adjoint local periodic problem. What is this equation in turbulence models????

EVOLUTION EQUATIONS AND SELF-SIMILARITY

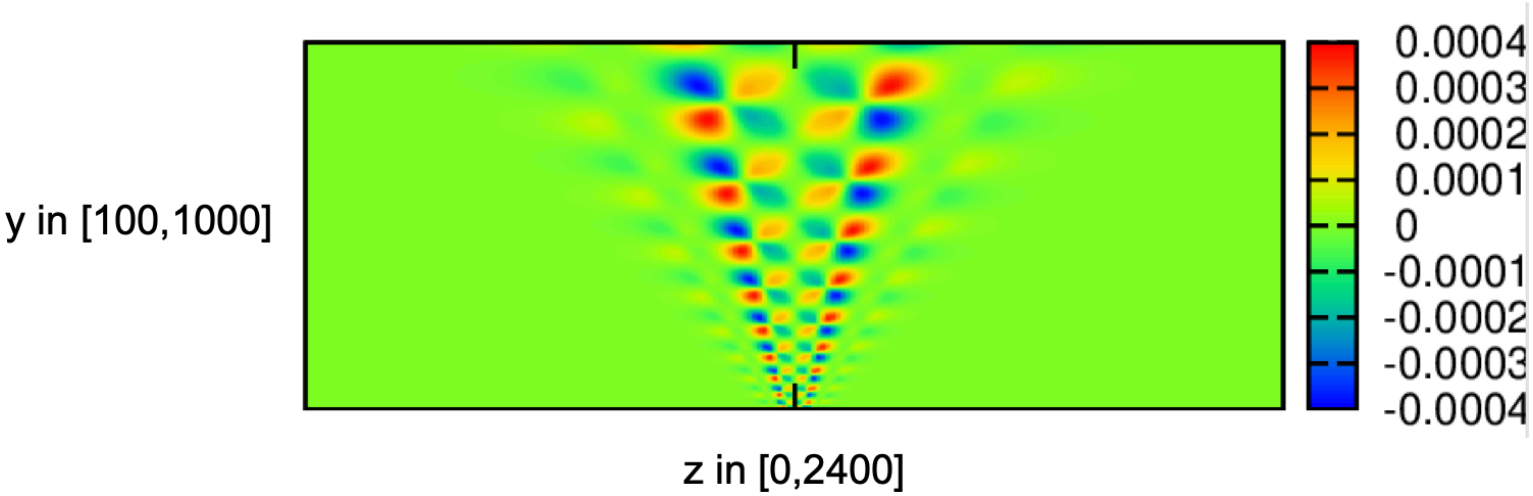
- For complete self similarity need $\Theta', \alpha \simeq Y^{-1}$ but spanwise wavelength is fixed.
- But VWI solutions localizing in the spanwise direction exist- this enables localization since they become independent of spanwise wavelength.
- The α equation allows solution only if $q \simeq O(1) as Y \to 0$. This fixes the unique streamwise wavelength allowing self-similarity.
- So localized solutions give spanwise extent of structure and streamwise wavelength all proportional to Y.

The equation for the mean then gives

$$\overline{u}' = \frac{N}{Y}, N = \beta\epsilon\sigma_0$$

so gives the Law of the Wall!!! but Von Karman constant depends on β, ϵ.

THE LOCALIZED SOLUTION MODE



SUMMARY OF RESULTS

- Solutions do possess uniform momentum zones.
- Solutions exist with complete self-similarity and imply the log law- but no unique Von Karman constant. Stuctures localizes in spanwise direction.
- ECS Can instead specify mean flow $\overline{u}$ and solve for α, θ'.- this predicts buffer layer and wake region.
- Lower branch states the key to drag reduction because they are 'almost' stable. (Alaskan pipeline)
- Upper branch solutions most relevant to DNS turbulent simulations